Beyond the Standard Model with $Sp(4)_c$ on the lattice

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arXiv: [2202.05191], [2304.07191], [2405.01388], [2405.05765], [2405.06506]

based on work with:
E. Bennett, L. Del Debbio, Y. Dengler, N. Forzano, D.K. Hong, R.C. Hill, H. Hsiao, S. Kulkarni, J.-W. Lee, C.-J. D. Lin, B. Lucini, A. Lupo, A. Maas, S. Mee, M. Nikolic, M. Piai, J. Pradler, F. Pressler, D. Vadacchino

[1] Holland et.al. [hep-lat/0312022], Mason et.al. [2310.02145] [4] Bennett et.al. [1712.04220]
[3] see e.g. Ferretti,Karateev [1312.5330], Ferretti [1604.06467], Hochberg et. al. [1402.5143] [1411.3727] [1512.07917], Kulkarni et.al. [2202.05191] Lattice: Bennett et.al. [1909.12662] [2304.07191] [2311.14663] [2312.08465], Dengler et.al. [2311.18549]

$Sp(4)$ Gauge Theories in BSM Models

- **QCD-like** extensions of the Standard Model:
- Studied in multiple variations on the lattice
  - Pure gauge theory:
    Deconfinement transition[1], glueballs & large-N[4]
  - With Fermions: **Higgs compositeness**, **Dark Matter** [3]

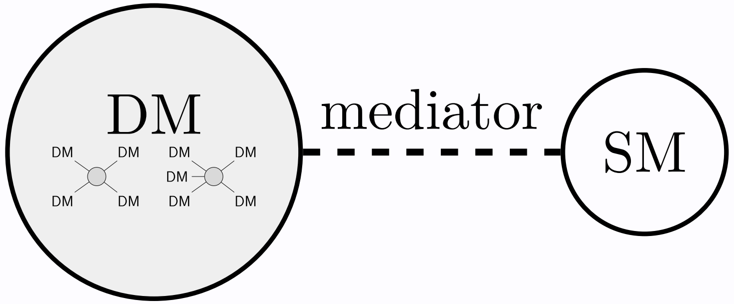
QCD-like gauge theories

- New non-SM gauge force with fermions
  - Composite Higgs Models: hyper-gluons and hyper-quarks
  - Dark Matter Models: dark gluons and dark quarks
- Depending on the BSM model they can carry SM charges or not

- I will use the QCD nomenclature:
  - e.g. π: 0^- nonsinglet, ρ: 1^- non-singlet, ...
  - these states are **not** the QCD hadrons
  - but they are similar in terms of their fermion structure

Strongly Interacting Dark Matter Models

- With fermions: Global symmetries make DM stable
- With mediator: Dark sector coupled to SM



$$\mathcal{L}_{\mathrm{DM}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}_f (i \not{D} + m_f) \psi_f$$

- Non-vanishing self-scattering cross-section arise

$$\langle v \sigma_{\pi\pi \to \pi\pi} \rangle \neq 0$$

- Relic density driven by strong processes see e.g. Ferretti,Karateev [1312.5330], Ferretti [1604.06467] Ayyar et.al. [1710.00806] [1801.05809] [1802.09644] Bergner,Piemonte [2008.02855][2111.15335] Cossu et. al. [1904.08885] Bennett et.al. [2202.05516] [2311.14663]

Composite Higgs From Multiple Fermion Representations

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu} + \bar{\mathrm{Q}}^{\mathrm{i}}\left(\mathrm{i}\not{D} - \mathrm{m}_{\mathrm{i}}^{\mathrm{f}}\right)\mathrm{Q}^{\mathrm{i}} + \bar{\Psi}^{\mathrm{j}}\left(\mathrm{i}\not{D} - \mathrm{m}_{\mathrm{j}}^{\mathrm{as}}\right)\Psi^{\mathrm{j}}$$

- Gauge theory of group G with field strength tensor $F_{\mu\nu}$
- Two species of fermions Q and Ψ under different irreps of G

`Applications:`

- **Composite Higgs+top**(fundamental and antisymmetric fermions)
- Models of supersymmetric physics (fundamental + adjoint)

[1] Kosower (Phys.Lett.B.144, 1984) Witten (Nucl.Phys.B.223, 1983) Peskin (Nucl.Phys.B.175, 1980)
[2] Witten (Nucl.Phys.B.149, 1979) (Nucl.Phys.B.156, 1979) Veneziano (Nucl.Phys.B.159, 1979)
[3] Belyaev et.al. [1512.07242]

Chiral Symmetry and Extra Goldstone Bosons

- One breaking pattern for every fermion representation [1]
  - complex: $SU(N_f) \times SU(N_f) \to SU(N_f)$
  - pseudoreal: $SU(2N_f) \to Sp(2N_f)$
  - real: $SU(2N_f) \to SO(2N_f)$
- And one axial $U(1)$ for each representation
  - one (combination of) $U(1)$ broken by axial anomaly! [2]
  - Additional $U(1)$ Goldstone for multiple representations! [3]
  - mixed state with contributions from different reps

Flavour symmetry: Pseudo-real representation

- Higher symmetry than theories with complex representations
  - same as two colour QCD $(SU(2) = Sp(2))$
- Mixing of left- and right-handed Weyl components

$$\Psi = \begin{pmatrix} u_L \\ d_L \\ -SCu_R^* \\ -SCd_R^* \end{pmatrix} = \begin{pmatrix} u_L \\ d_L \\ \tilde{u}_R \\ \tilde{d}_R \end{pmatrix} \qquad \begin{matrix} C \ldots \text{charge conj.} \\ S \ldots \text{colour matrix} \end{matrix}$$

$$\mathcal{L}_{\text{fermion}} = i\bar{\Psi}\not{D}\Psi - \frac{1}{2}\left(\Psi^T SCM\Psi + h.c.\right)$$

- Mass matrix M proportional to symplectic invariant tensor
- generators $\tau_a : S\tau_a S = -\tau_a^T$

The Hadron Spectrum of $Sp(4)$ Theories

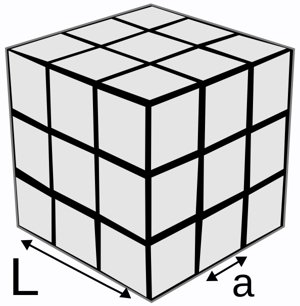
1. $Sp(4)_c$ with $N_f = 2$ (fundamental)
2. $Sp(4)_c$ with $N_f = 2$ (fundamental) and $n_f = 3$ (antisymmetric)

Lattice setup

- Euclidean action S on hypercubic lattice

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[A_\mu, \psi, \bar{\psi}] e^{-S[A_\mu, \psi, \bar{\psi}]} O[A_\mu, \psi, \bar{\psi}]$$

- Lattice regulator: finite spacing a (UV), finite extent L (IR)



- Calculate observable $\langle O \rangle$ on finite lattice
- Extrapolate to the continuum: $a \to 0$, $L \to \infty$
- Wilson fermions, GRID code on GPUs, HiRep code on CPUs

Other mass scales than QCD are potentially relevant!

- Free parameters: Coupling g^2 and bare fermion masses
  - one overall energy scale
  - one fermionic mass scale for every representation
- All scales can deviate strongly from QCD!
  - can result in different meson mass hierarchies
  - can give rise to a different finite-temperature phase diagram

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Lattice investigations of a larger parameter space are useful!
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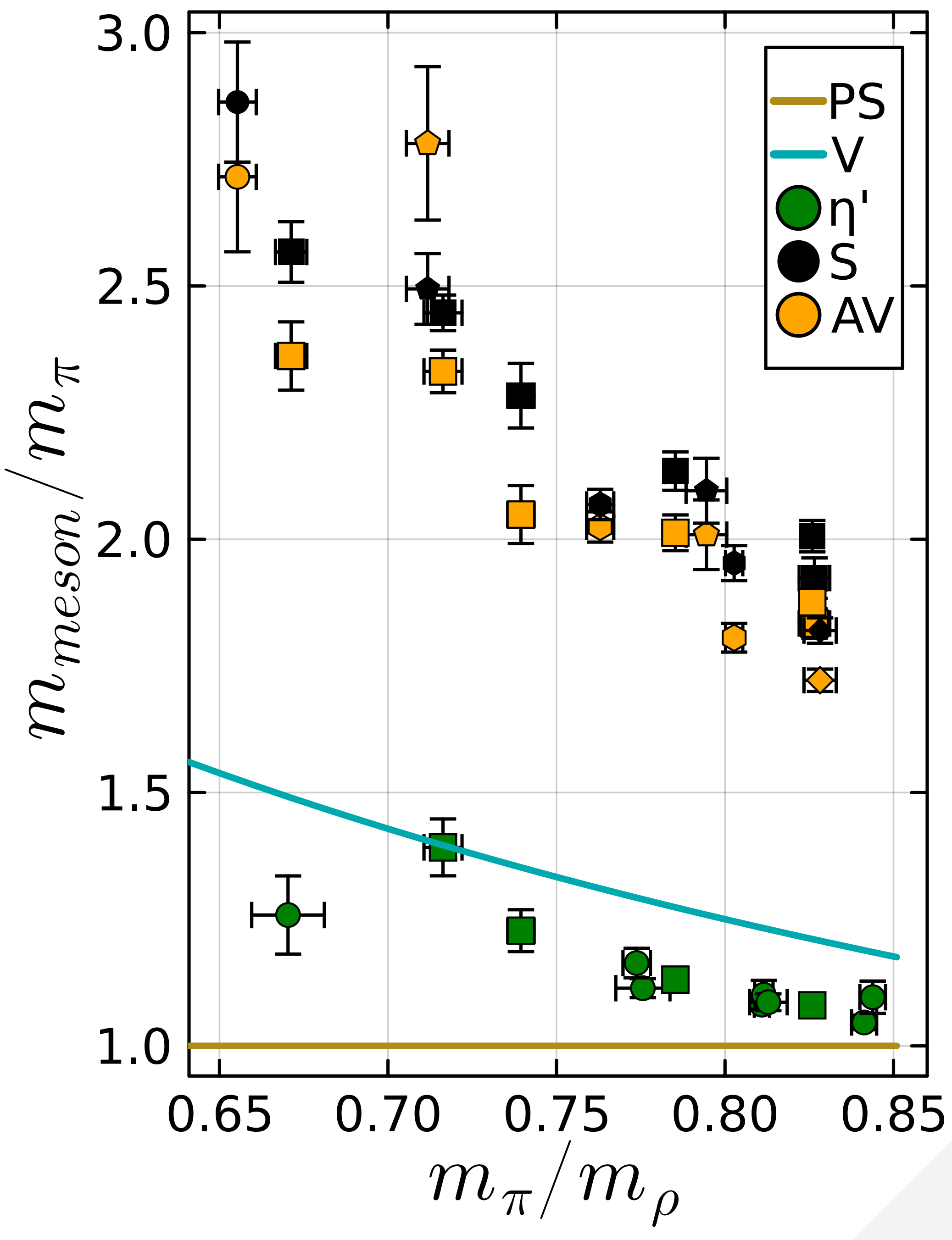
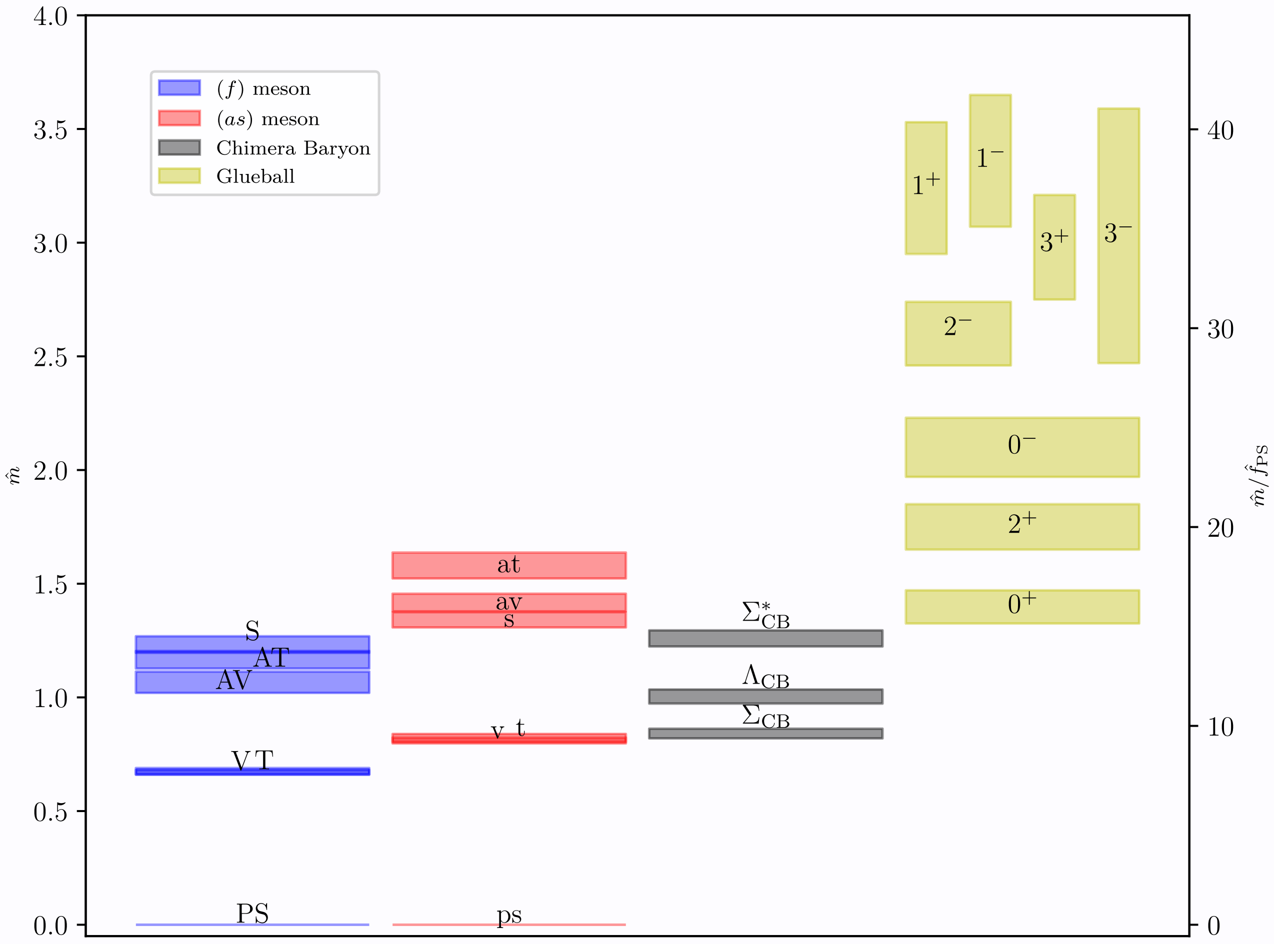
The $N_f = 2$ spectrum of $Sp(4)$

- Strongly resembles QCD
- Most notable difference: η'
  - similar to two-flavour QCD
- $\pi\pi$-scattering investigated (see poster by Y. Dengler)
- f_0/σ and glueballs under ongoing investigation



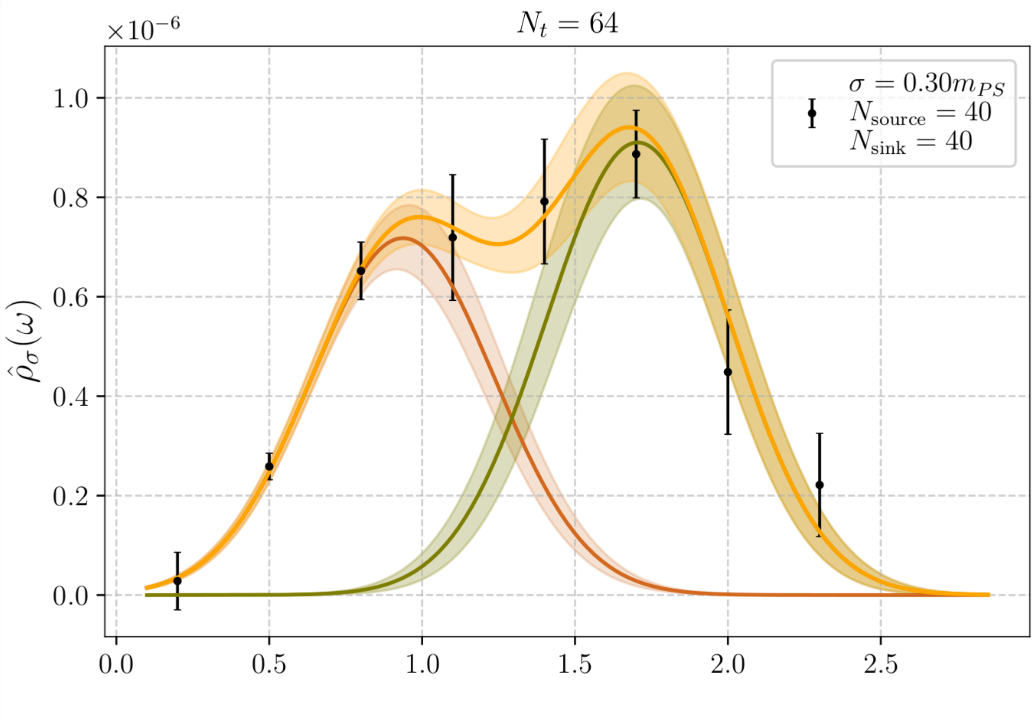
The spectrum of $Sp(4)$ with mixed representation

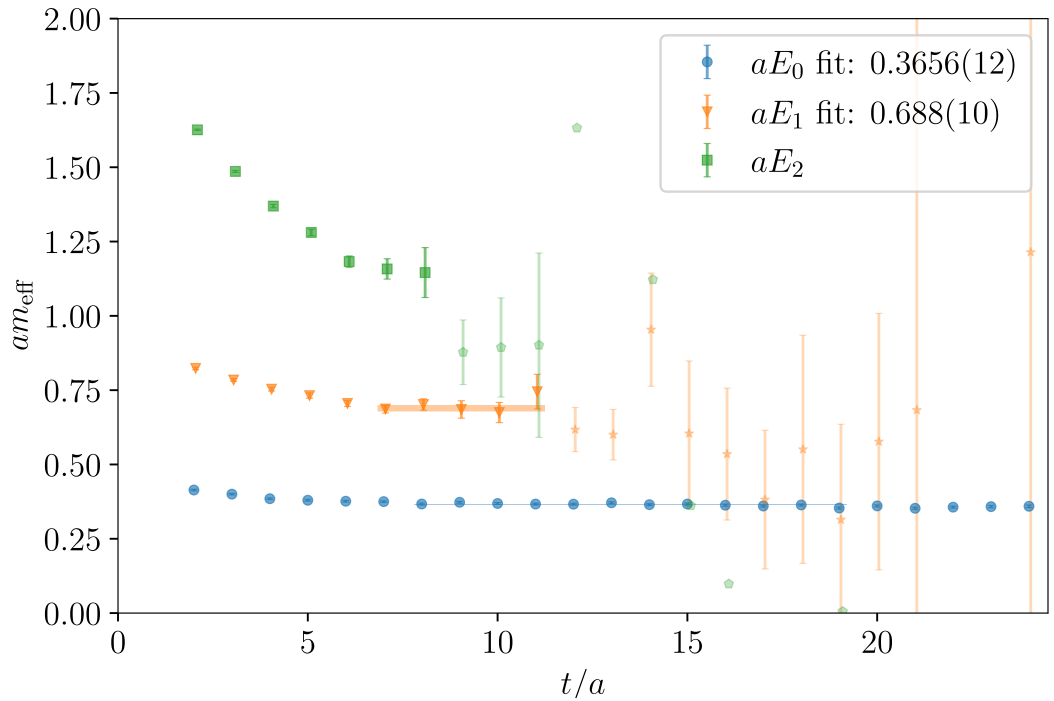


- Two fundamental $N_f = 2$ and three antisymmetric $n_f = 3$ fermions
- Quenched hadron and glueball masses are available [1]
- But we need to go towards **dynamical fermions**

Mixed-representations: With dynamical fermions

- Non-singlet meson masses determined using variational analysis and from spectral densities [1]





[1] e.g. Belyaev et.al. [1610.06591], Cacciapaglia et.al. [1902.06890] Franzosi et.al. [2106.12615]

[2] DeGrand et.al. [1605.07738]

Mixed-representations: States connected to $U(1)_A$

- pseudoscalar flavour-singlets: similar to η and η' of QCD
- **Potentially light singlet can have large pheno implications!** [1]

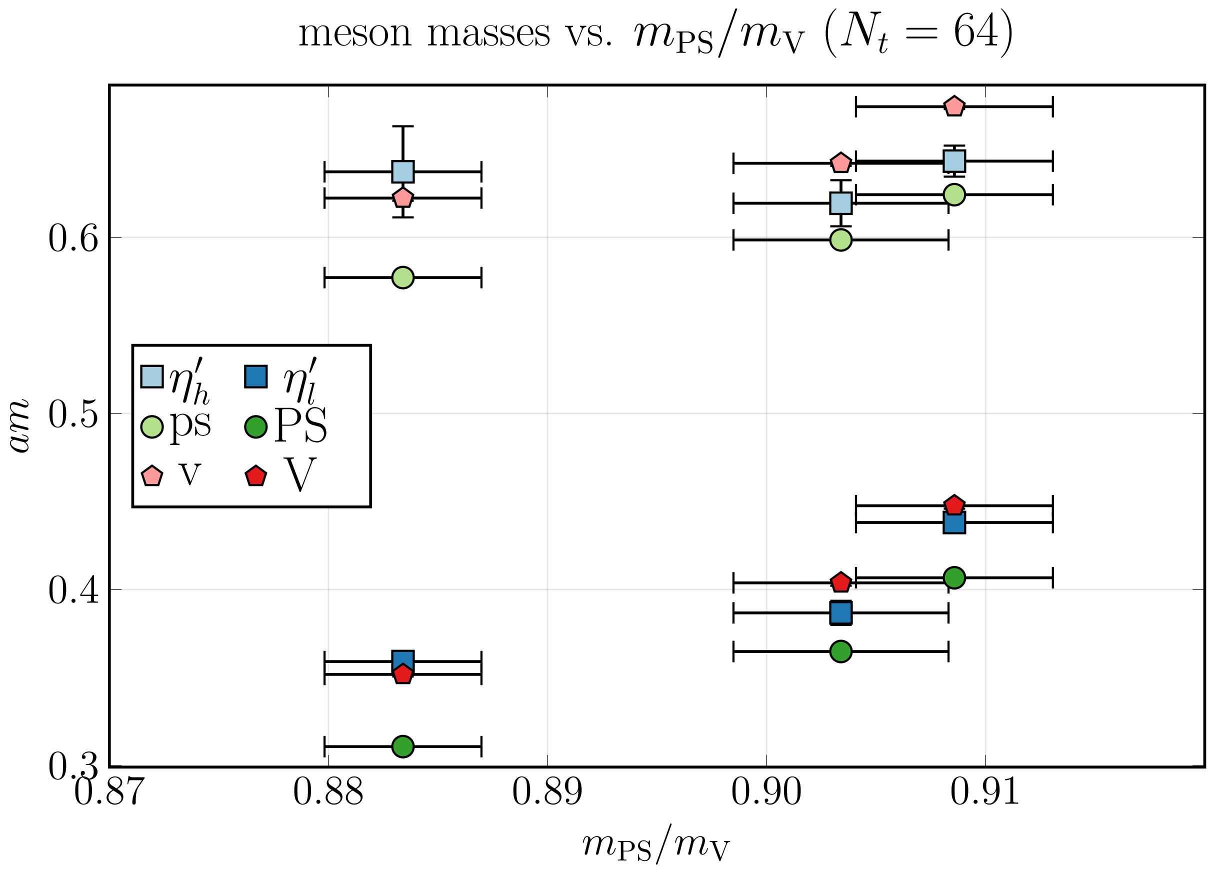
$$O_{\eta^{\mathrm{f}}} = \left(\bar{Q}^1\gamma_5 Q^1 + \bar{Q}^2\gamma_5 Q^2\right)/\sqrt{2}$$

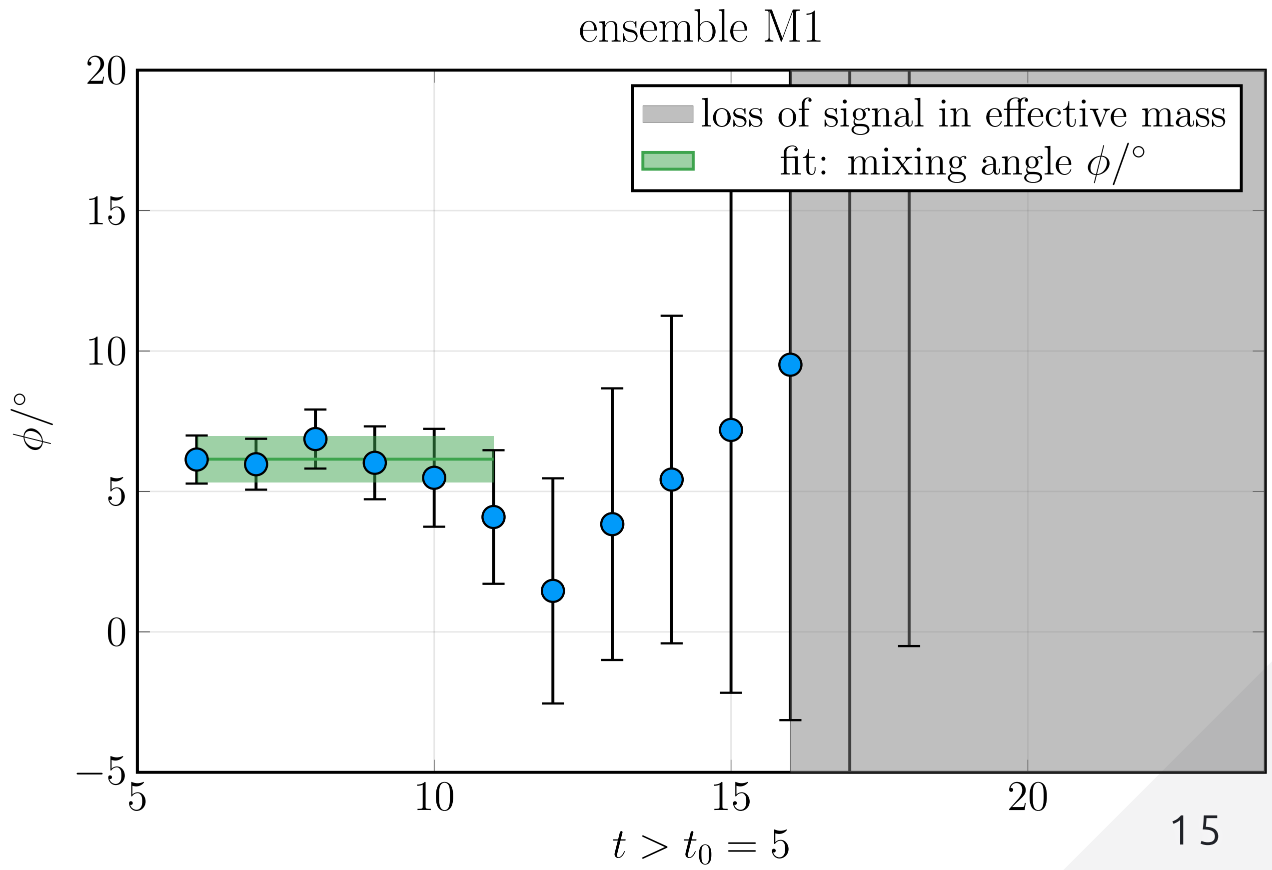
$$O_{\eta^{\mathrm{as}}} = \left(\bar{\Psi}^1\gamma_5 \Psi^1 + \bar{\Psi}^2\gamma_5 \Psi^2 + \bar{\Psi}^3\gamma_5 \Psi^3\right)/\sqrt{3}$$

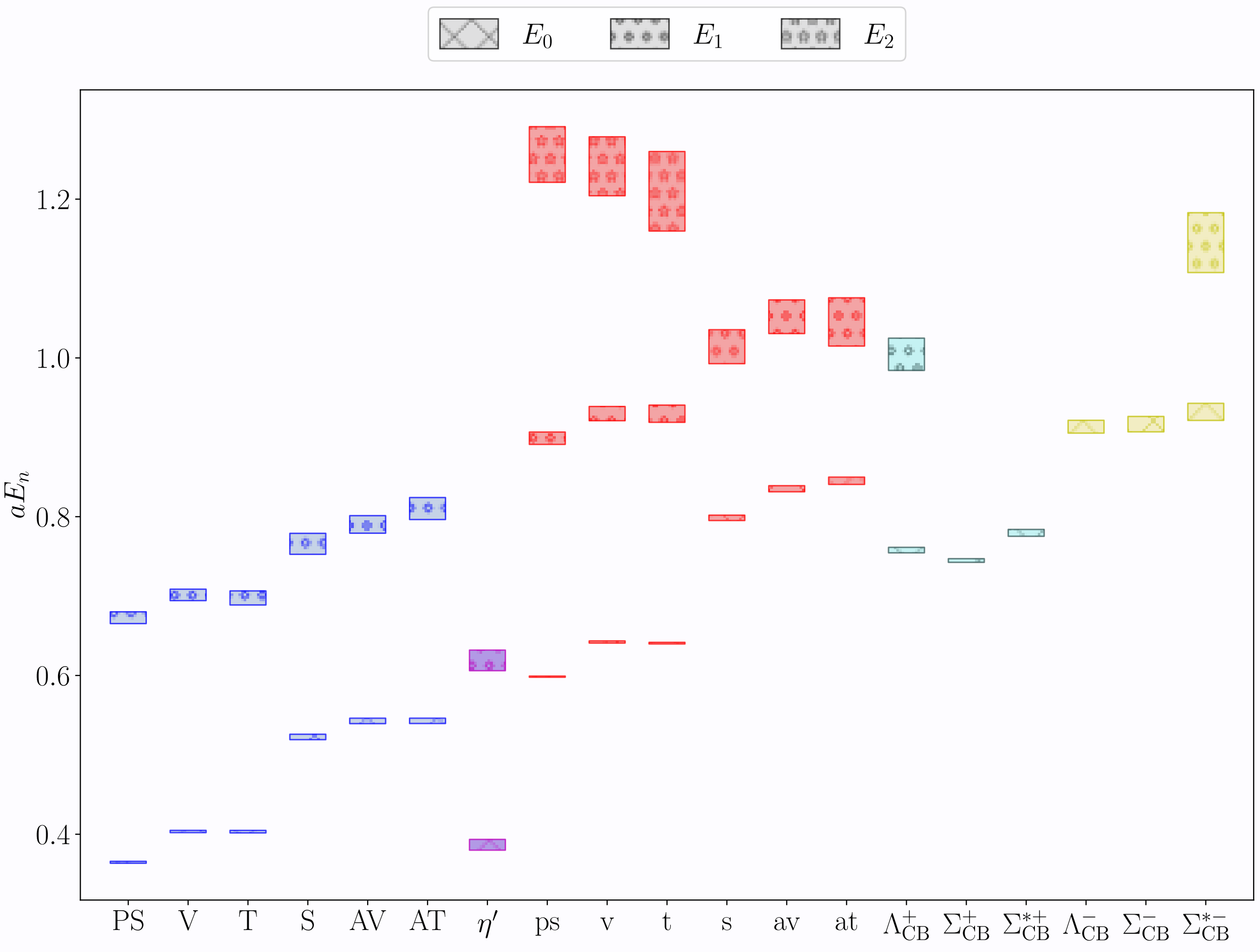
- These two states will mix: Light PNGB state η'_l + heavier state η'_h
  - mixing angle in general $\phi \neq 0$
  - Effective field theory in chiral limit has been developed [2]

Mixed-representations: States connected to $U(1)_A$

- Masses close to respective non-singlet states, small mixing angle
  - fermion masses appear to be far from chiral limit







Baryon spectrum of single ensemble

- Preliminary determination of hybrid baryons ($QQ\psi$)
- Plot depicts single lattice ensemble
- Glueballs and scalar singlet under investigation

Looking forward: Connection to the phase diagram

- Pure Gauge: Known first-order deconfinement transition [1]
  - Recently investigated using density-of-states method LLR [2]
  - Goal: Access to gravitational wave parameters
  - *Theories with Fermions require further algorithmic development*
- Lattice Simulations at finite density possible
  - No sign problem (as in QC_2D)

Summary

- $Sp(2N)$ (and QCD-like theories in general) for BSM scenarios
- **Zero-temperature results for $Sp(4)$ available and improving!**
- Single rep.: Moderately light fermions available, chiral extrapolation feasible
- Mixed rep.: Currently restricted to heavy dynamical fermions

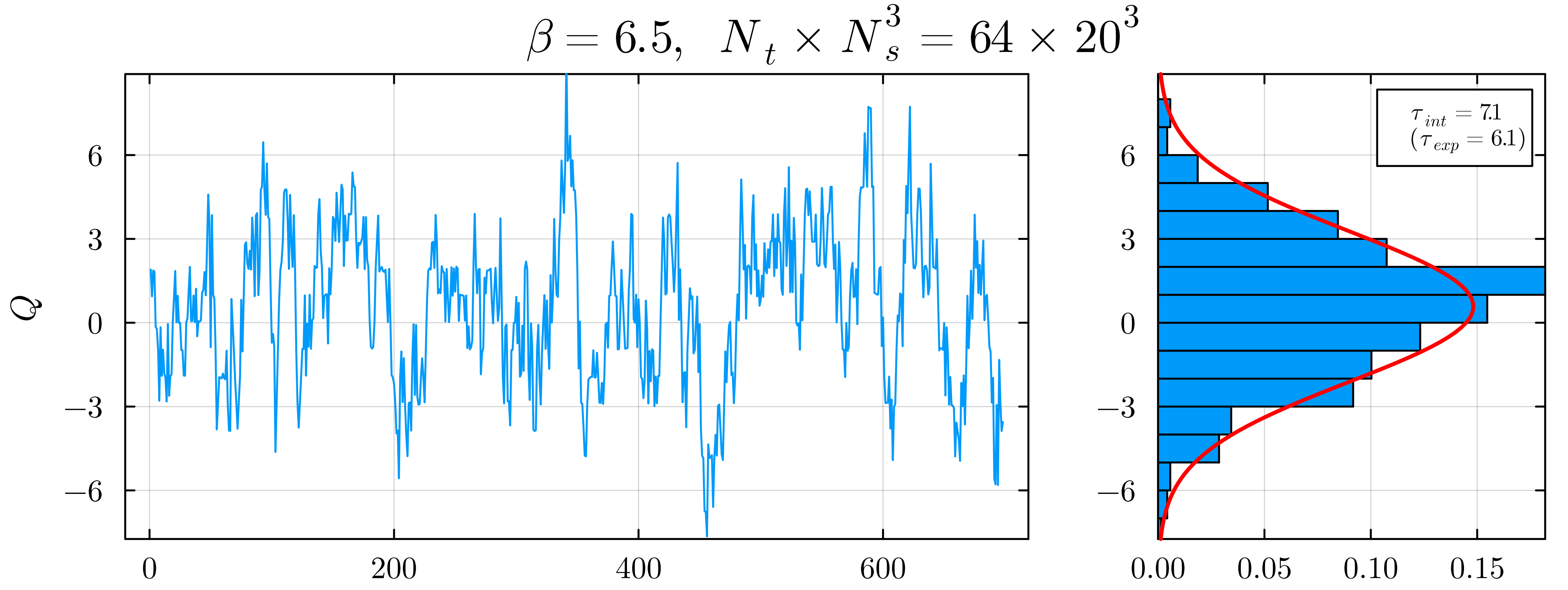
Outlook

- Finer lattice spacings, lighter fermions, different discretizations
- Glueballs, scalar singlet channel, and more mixing
- Meson scattering and resonances

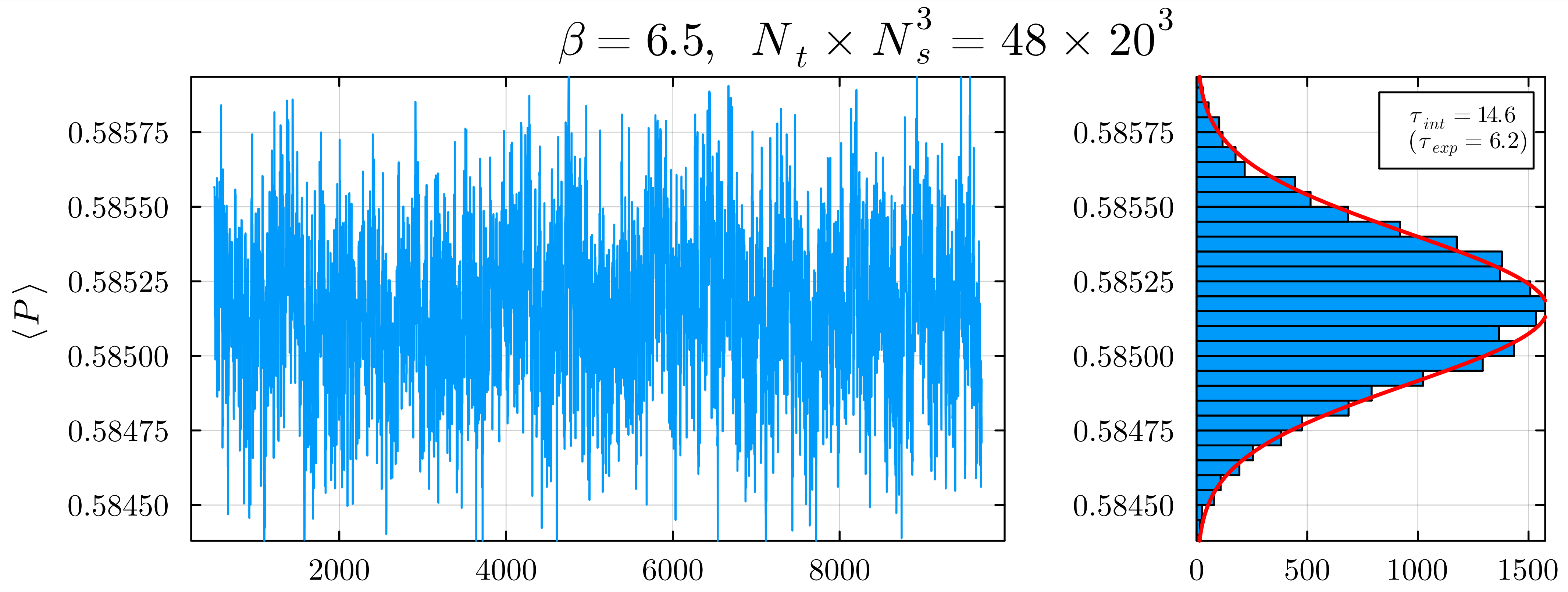
Back-up slides

Topology of ensembles

- not frozen, but correlations in topological charge Q



Configurations chosen such that plaquette is uncorrelated



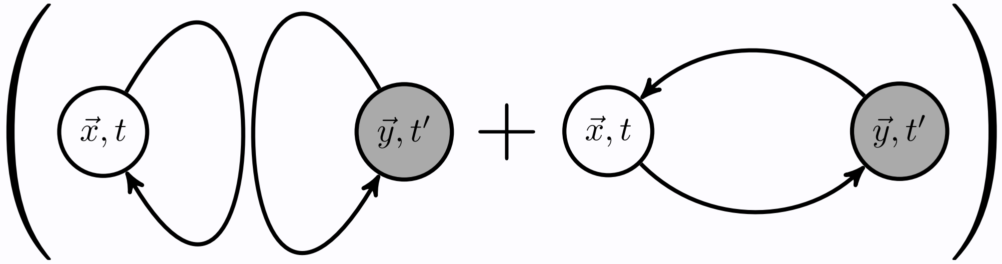
Lattice spectroscopy: Getting meson masses

- Construct operator with same quantum numbers
- Energy levels from Euclidean correlator

$$C_{\mathcal{O}}(t) = \sum_n \frac{1}{2E_n} \langle 0|\mathcal{O}|n\rangle^* \langle n|\mathcal{O}|0\rangle e^{-E_n t}.$$

- For mesons a generic correlator

$$C(t-t') = \sum_{\vec{x},\vec{y}} \left(\ldots \right) + \underbrace{\text{const.}}_{=|\langle 0|O|0\rangle|^2}$$



Available Dynamical Ensembles

Label	β	am_0^{as}	am_0^{f}	N_t	N_s	N_{conf}
M1	6.5	-1.01	-0.71	48	20	479
M2	6.5	-1.01	-0.71	64	20	698
M3	6.5	-1.01	-0.71	96	20	436
M4	6.5	-1.01	-0.70	64	20	709
M5	6.5	-1.01	-0.72	64	32	295

- Ensembles generated using GPUs with GRID
- Measurements performed with HiRep on CPUs

table taken from Feretti [1604.06467], see also Feretti,Karateev [1312.5330]

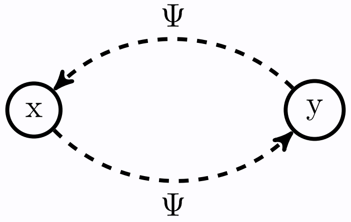
Composite Higgs + Top Realizations

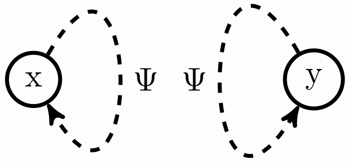
$G_{\rm HC}$	ψ	χ	Restrictions	G/H
$\mathrm{SO}(N_{\rm HC})$	$5\times\mathbf{F}$	$6\times\mathbf{Spin}$	$N_{\rm HC}=7,9$	$\frac{\mathrm{SU}(5)}{\mathrm{SO}(5)}\,\frac{\mathrm{SU}(6)}{\mathrm{SO}(6)}\,\mathrm{U}(1)$
$\mathrm{SO}(N_{\rm HC})$	$5\times\mathbf{Spin}$	$6\times\mathbf{F}$	$N_{\rm HC}=7,9$	
$\mathrm{Sp}(2N_{\rm HC})$	$5\times\mathbf{A}_2$	$6\times\mathbf{F}$	$2N_{\rm HC}=4$	$\frac{\mathrm{SU}(5)}{\mathrm{SO}(5)}\,\frac{\mathrm{SU}(6)}{\mathrm{Sp}(6)}\,\mathrm{U}(1)$
$\mathrm{SU}(N_{\rm HC})$	$5\times\mathbf{A}_2$	$3\times(\mathbf{F},\overline{\mathbf{F}})$	$N_{\rm HC}=4$	$\frac{\mathrm{SU}(5)}{\mathrm{SO}(5)}\,\frac{\mathrm{SU}(3)\times\mathrm{SU}(3)'}{\mathrm{SU}(3)_D}\,\mathrm{U}(1)$
$\mathrm{SO}(N_{\rm HC})$	$5\times\mathbf{F}$	$3\times(\mathbf{Spin},\overline{\mathbf{Spin}})$	$N_{\rm HC}=10$	
$\mathrm{Sp}(2N_{\rm HC})$	$4\times\mathbf{F}$	$6\times\mathbf{A}_2$	$2N_{\rm HC}=4$	$\frac{\mathrm{SU}(4)}{\mathrm{Sp}(4)}\,\frac{\mathrm{SU}(6)}{\mathrm{SO}(6)}\,\mathrm{U}(1)$
$\mathrm{SO}(N_{\rm HC})$	$4\times\mathbf{Spin}$	$6\times\mathbf{F}$	$N_{\rm HC}=11$	
$\mathrm{SO}(N_{\rm HC})$	$4\times(\mathbf{Spin},\overline{\mathbf{Spin}})$	$6\times\mathbf{F}$	$N_{\rm HC}=10$	$\frac{\mathrm{SU}(4)\times\mathrm{SU}(4)'}{\mathrm{SU}(4)_D}\,\frac{\mathrm{SU}(6)}{\mathrm{SO}(6)}\,\mathrm{U}(1)$
$\mathrm{SU}(N_{\rm HC})$	$4\times(\mathbf{F},\overline{\mathbf{F}})$	$6\times\mathbf{A}_2$	$N_{\rm HC}=4$	
$\mathrm{SU}(N_{\rm HC})$	$4\times(\mathbf{F},\overline{\mathbf{F}})$	$3\times(\mathbf{A}_2,\overline{\mathbf{A}}_2)$	$N_{\rm HC}=5,6$	$\frac{\mathrm{SU}(4)\times\mathrm{SU}(4)'}{\mathrm{SU}(4)_D}\,\frac{\mathrm{SU}(3)\times\mathrm{SU}(3)'}{\mathrm{SU}(3)_D}\,\mathrm{U}(1)$

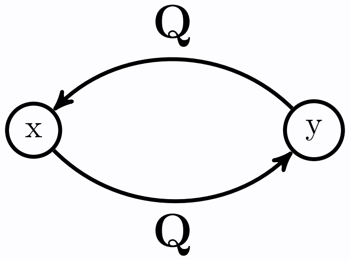
Table 6. Subclass of models that is likely to be outside of the conformal window, together with the coset they give rise to after spontaneous symmetry breaking.

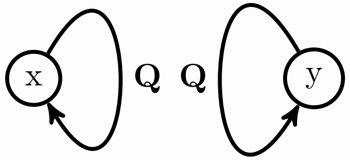
Lattice Investigation: Masses

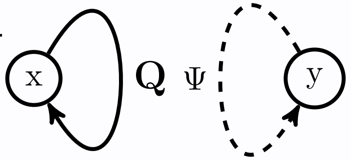
- Variational Analysis with $O_{\eta^{\mathrm{f}}}$ and $O_{\eta^{\mathrm{as}}}$ operators
- Several levels of Wuppertal smearing

$\langle \bar{O}_{\eta^{\mathrm{as}}}(x) O_{\eta^{\mathrm{as}}}(y) \rangle = -$ 

$+ N_{\mathrm{as}}$ 

$\langle \bar{O}_{\eta^{\mathrm{f}}}(x) O_{\eta^{\mathrm{f}}}(y) \rangle = -$ 

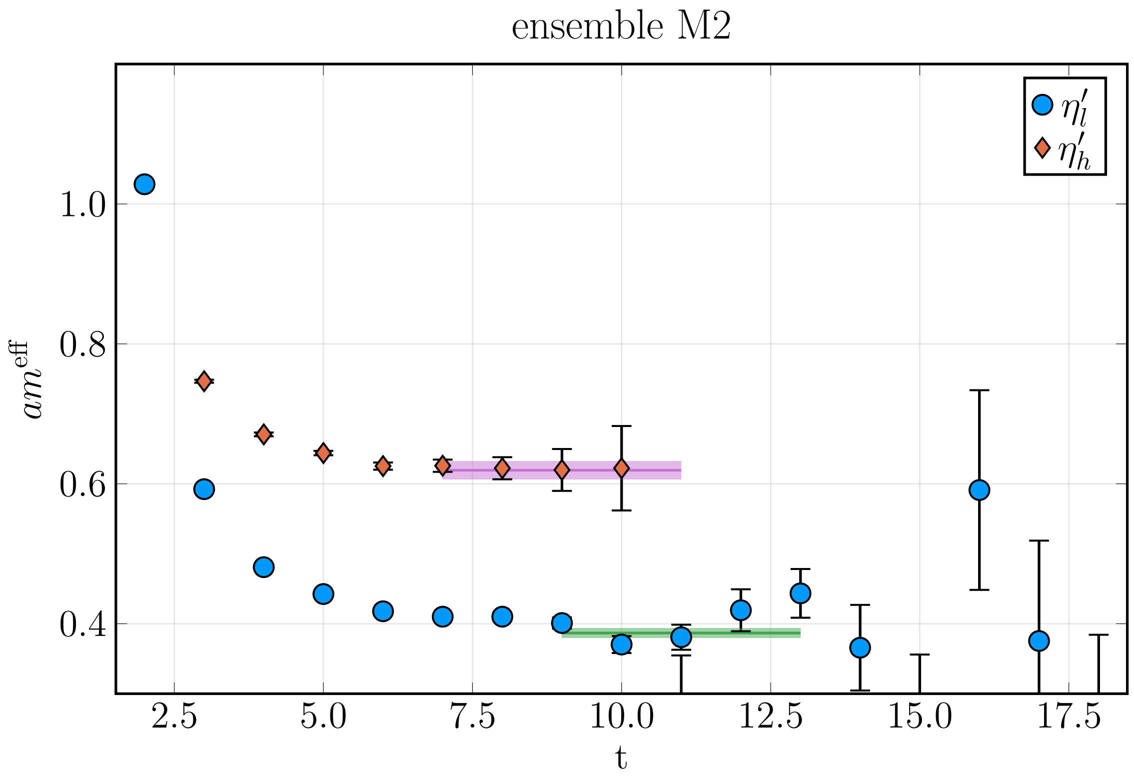
$+ N_{\mathrm{f}}$ 

$\langle \bar{O}_{\eta^{\mathrm{f}}}(x) O_{\eta^{\mathrm{as}}}(y) \rangle = +\sqrt{N_{\mathrm{as}} N_{\mathrm{f}}}$ 

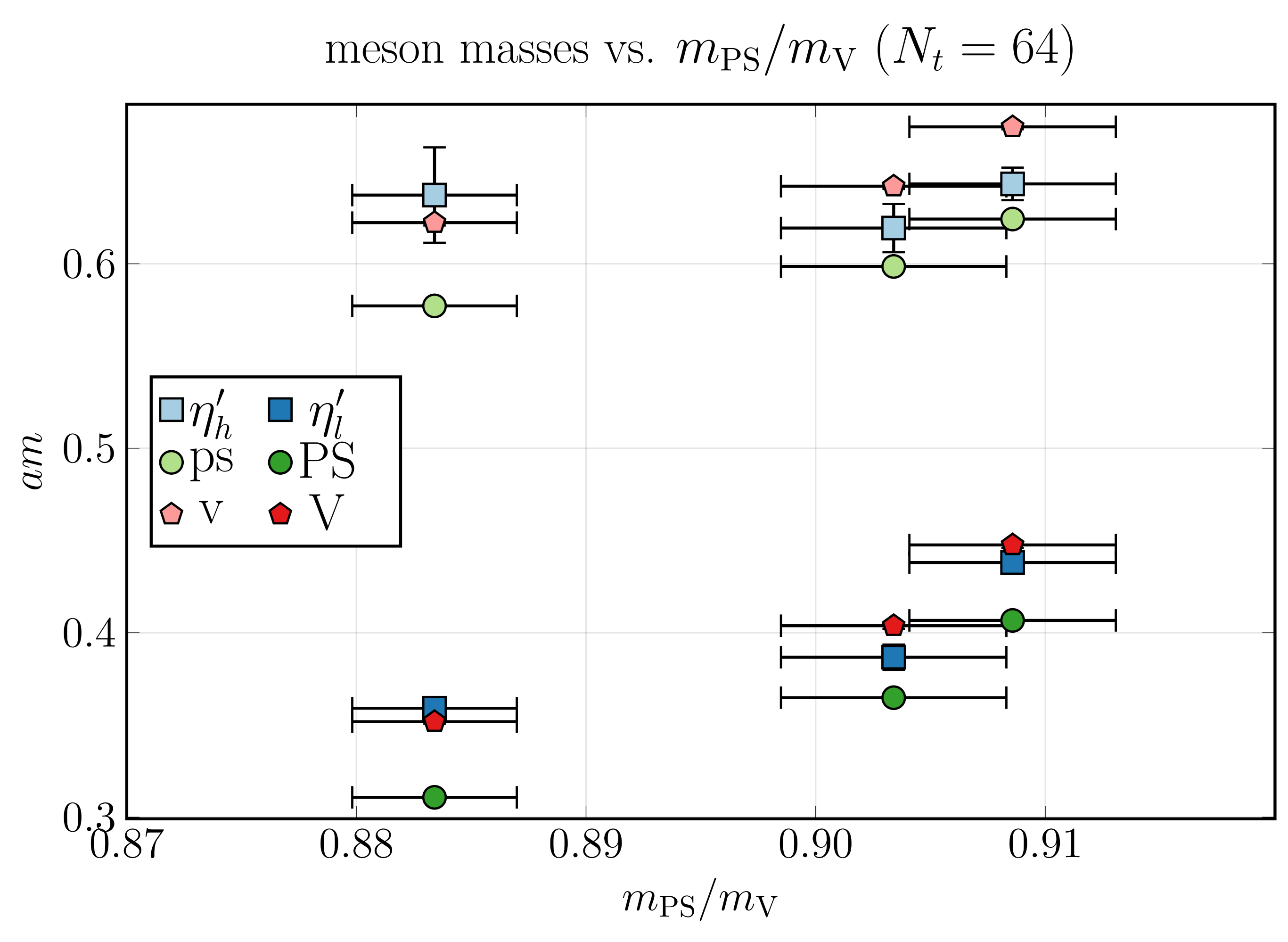
- N_{f} and N_{as} enhance singlet contributions
- non-vanishing fermion masses $(m_{\mathrm{f}}, m_{\mathrm{as}})$ suppress them

Results: Effective masses for η_l' and η_h'

- Variational analysis for correlation matrix $C_{ij}(t)$
- n^{th} eigenvalue falls off exponentially with energy E_n
- Masses from fits to correlator at large t

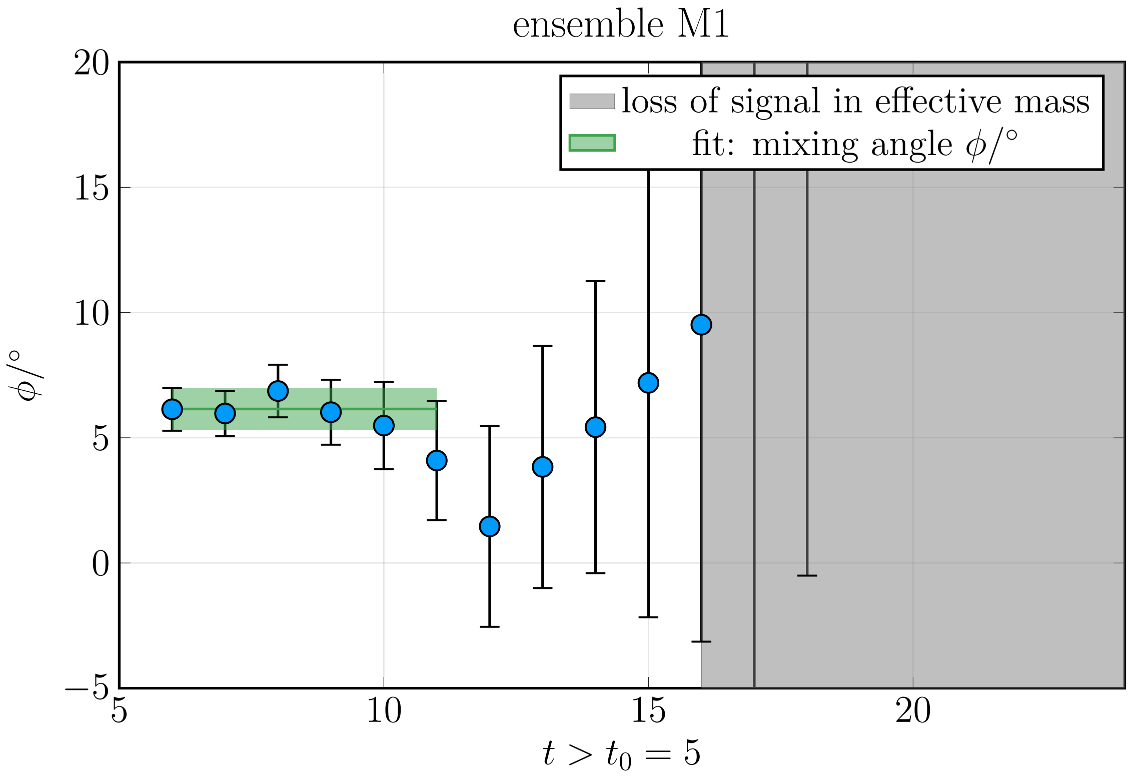


Results: Pseudoscalar Singlet Masses



- Spectrum likely dominated by heavy fermion masses

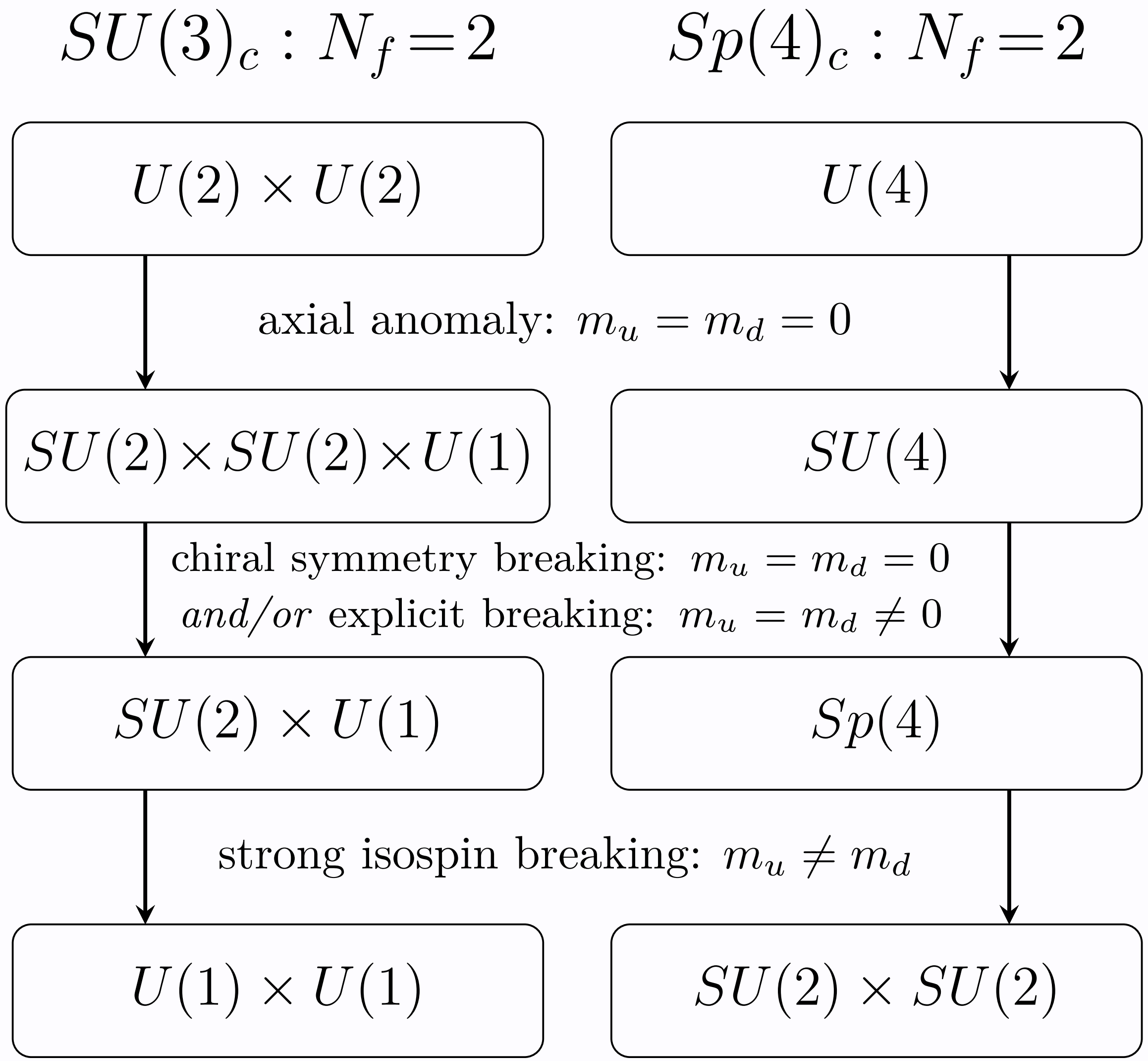
Results: Mixing Angle ϕ small



- Consistently small mixing angles

Label	β	N_t	N_s	$\phi/°$
M1	6.5	48	20	**6.15(83)**
M2	6.5	64	20	**6.07(63)**
M3	6.5	96	20	**6.16(66)**
M4	6.5	64	20	**7.44(58)**
M5	6.5	64	32	**6.61(54)**

[1] Kosower (Phys.Lett.B. 1984)
[2] Hochberg et. al. [1411.3727] [1512.07917]



SIMPs from $Sp(4)$ gauge theory

- Pseudo-real representation: [1]
  $\Rightarrow$ more pseudo-Goldstones
  $\Rightarrow$ no fermionic bound states
- $N_f = 2$: exactly 5 Goldstones
  - Allows $3\text{DM} \rightarrow 2\text{DM}$ [2]

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Sp(4) with two fermions is
a minimal SIMP DM realisation
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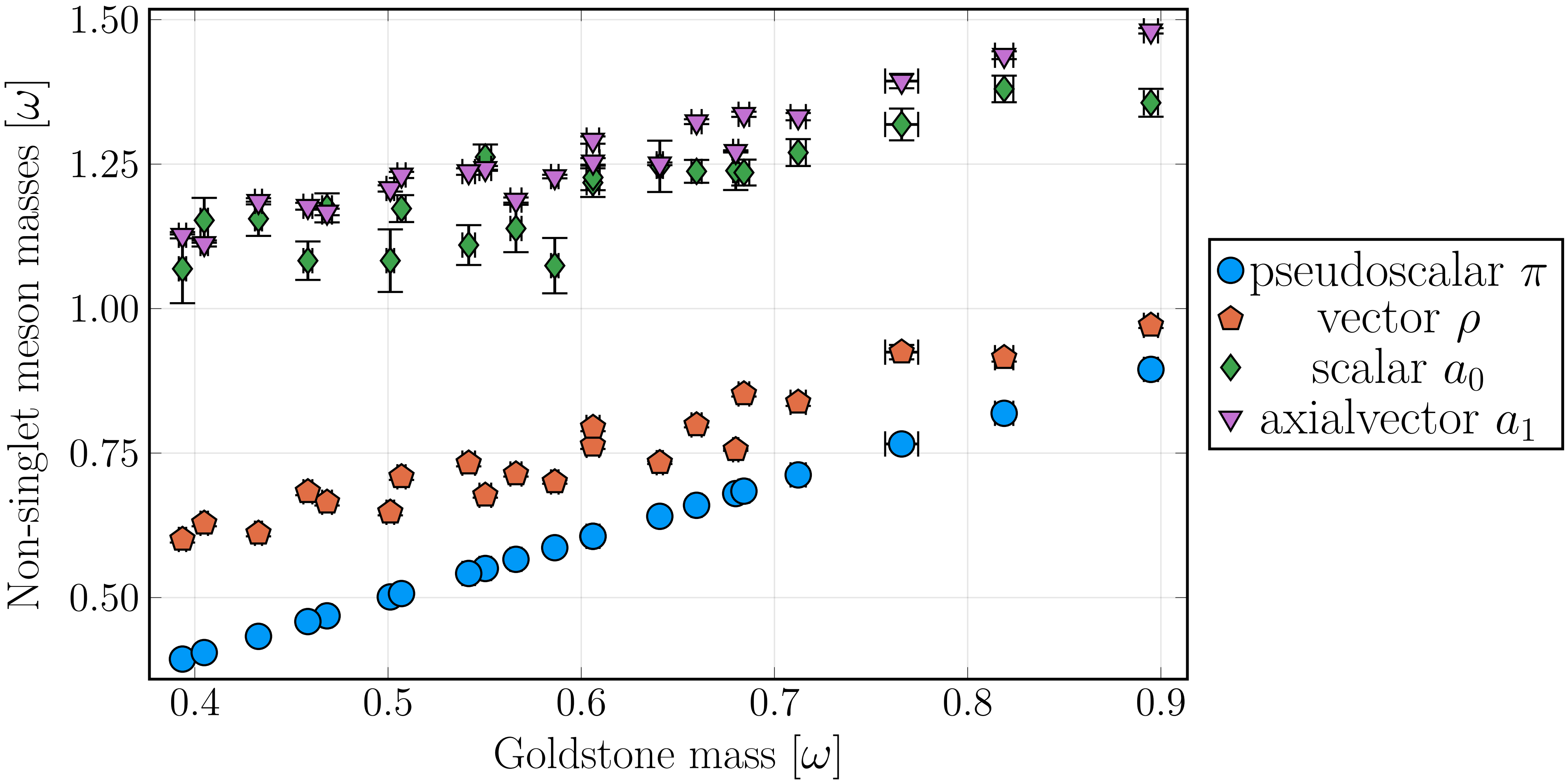
Non-singlet spectrum

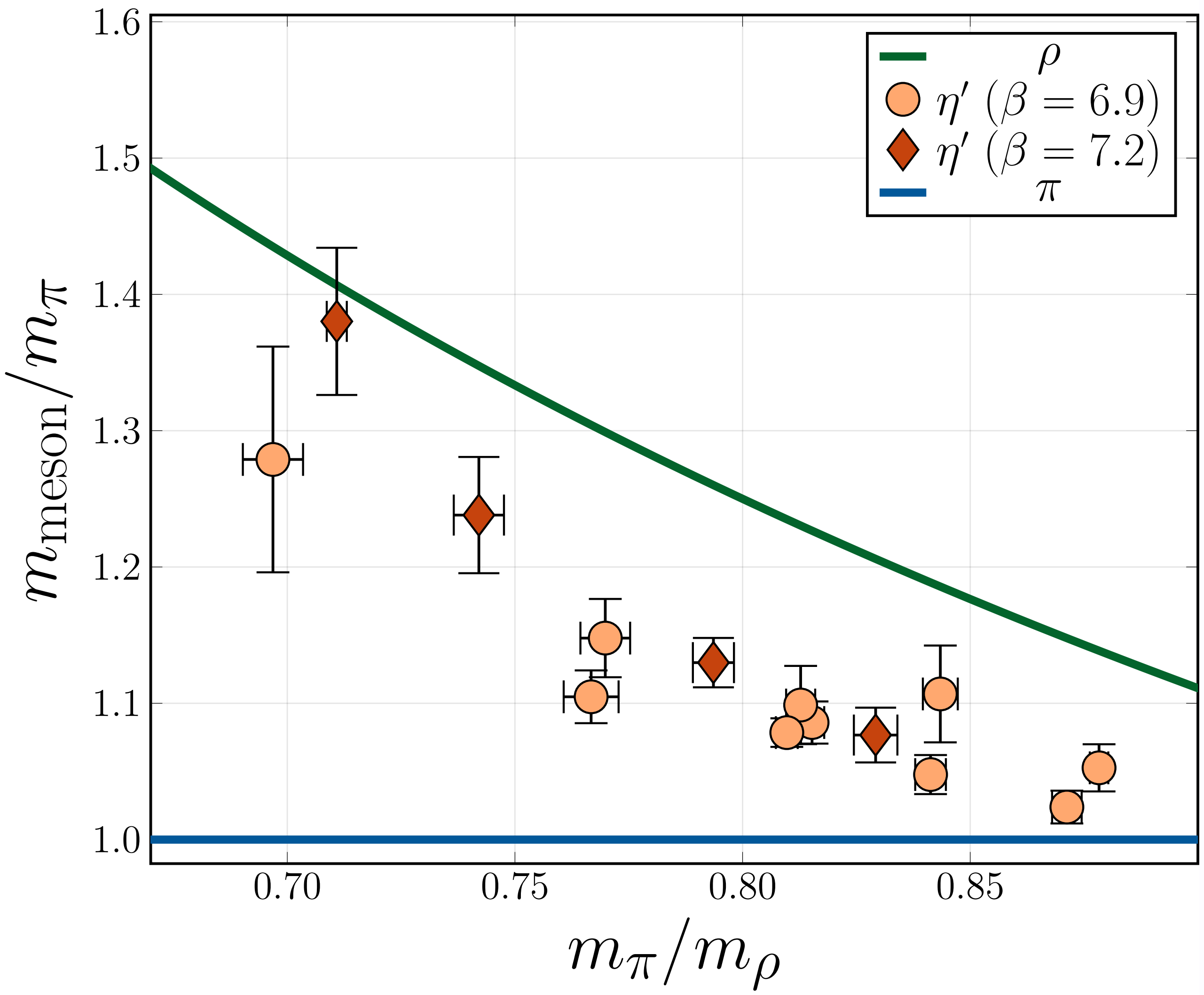
Bennett et. al. [1909.12662]



The pseudoscalar and vector mesons are the lightest non-singlets.

[1] Bennett et. al. [2304.07191]

The pseudoscalar singlet η' is surprisingly light!



- Phenomenologically relevant:
  - $m_\rho > m_{\eta'}$ different from QCD
  - relevant low-energy dof
  - η' relevant for $\pi\pi$ scattering
  - more accessible channels for decays into SM

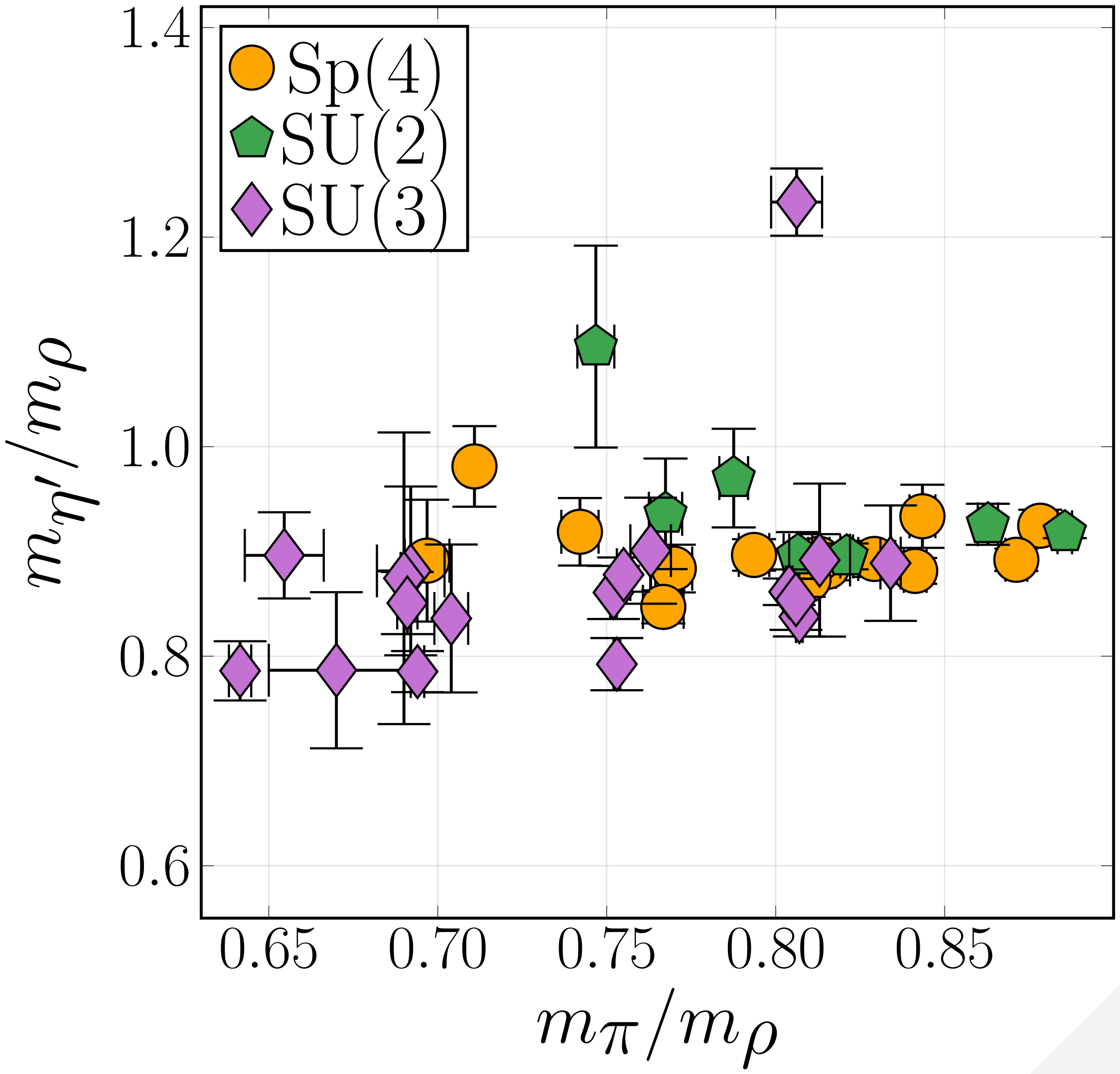
Interesting! Is this surprising?

Bennett et. al. [2304.07191]

Consider different theories:

- Large N_c: $m_{\eta'} - m_\pi \propto N_f / N_c$
  - $N_f = 2$ could be "small"
  - $N_c = 4$ could be "large"

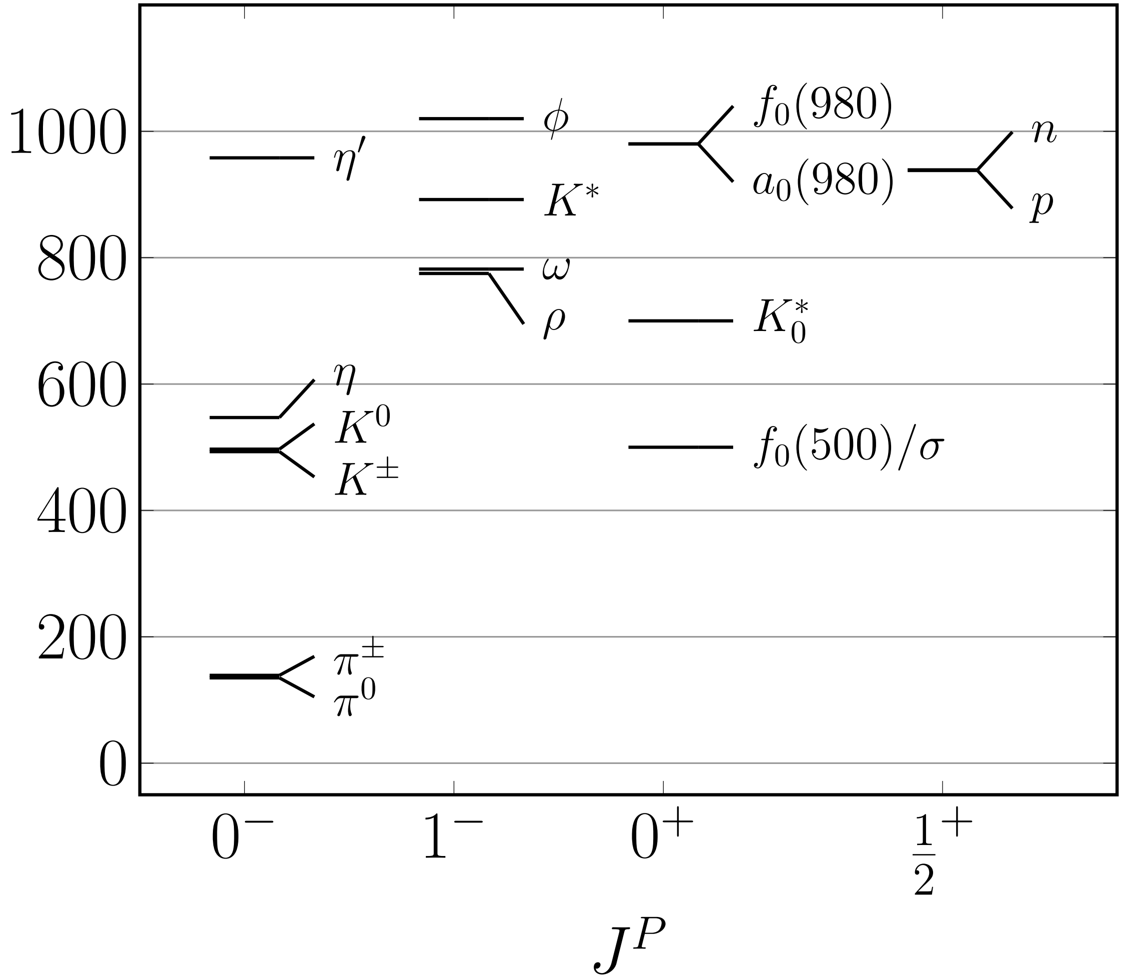
SU(2) and SU(3) comparison:



- Similarities:generic $N_f = 2$ feature?
- QCD: strong N_f dependence
- Differences may arise $m_\pi / m_\rho \to 0$

mass driven by flavour content!

Experimental light hadron masses [MeV]



QCD Spectrum

- π, K, η light: pseudo-Goldstones
- Vectors and scalars light
- Light and broad 0^+ singlet f_0/σ
- Heavy 0^- singlet η'
  $\Rightarrow U(1)_A$ anomalously broken

[1] Feldmann, Kroll, Stech [hep-ph/9802409]

Lattice Investigation: Mixing Angle

- Obtained from operator mixing (no signal for decay constants)
- Use of flavour basis justifies use of single mixing angle [1]

$$\begin{pmatrix} \langle 0|O_{\eta^{\mathrm{f}}}|\eta_l'\rangle & \langle 0|O_{\eta^{\mathrm{as}}}|\eta_l'\rangle \\ \langle 0|O_{\eta^{\mathrm{f}}}|\eta_h'\rangle & \langle 0|O_{\eta^{\mathrm{as}}}|\eta_h'\rangle \end{pmatrix} = \begin{pmatrix} A_{\mathrm{f}}^{\eta_l'} & A_{\mathrm{as}}^{\eta_l'} \\ A_{\mathrm{f}}^{\eta_h'} & A_{\mathrm{as}}^{\eta_h'} \end{pmatrix} \equiv \begin{pmatrix} A_{\eta_l'}\cos\phi & A_{\eta_l'}\sin\phi \\ -A_{\eta_h'}\sin\phi & A_{\eta_h'}\cos\phi \end{pmatrix}$$

- Matrix elements are obtained from the eigenvectors of the GEVP
- Expected to be constant for all timeslices t
- Test for dominance of fermion masses:
  - $m_{\mathrm{fermions}} \to \infty$ implies that $\phi \to 0$

An example: $Sp(4)$ with 2 fundamental + 3 antisymmetric

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu} + \bar{\mathrm{Q}}^{\mathrm{i}}\left(\mathrm{i}\not{D} - \mathrm{m}_{\mathrm{i}}^{\mathrm{f}}\right)\mathrm{Q}^{\mathrm{i}} + \bar{\Psi}^{\mathrm{j}}\left(\mathrm{i}\not{D} - \mathrm{m}_{\mathrm{j}}^{\mathrm{as}}\right)\Psi^{\mathrm{j}}$$

- **Non-perturbative input needed for pheno $\Rightarrow$ Lattice**
- $5 + 20 + 1$ pseudo-Goldstones + 1 $U(1)_A$ state
- The two $U(1)$ states will mix: both are 0^- iso-singlets
- Fermionic bound states $QQ\Psi$ provide top partner

- **Goals: Determine hadron spectrum on the lattice**
- In parallel: Develop techniques for finite temperature!