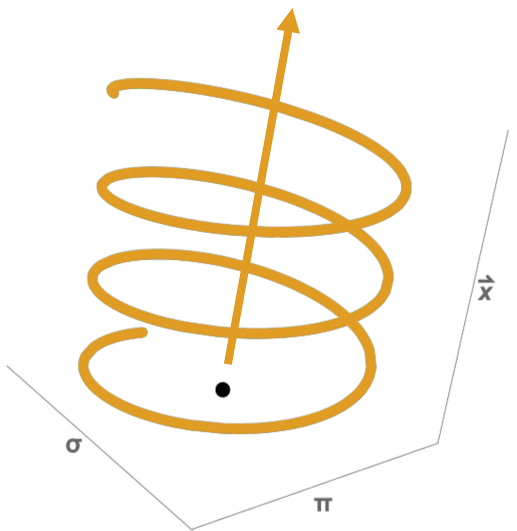
Chiral crossover vs chiral density wave in dense nuclear matter

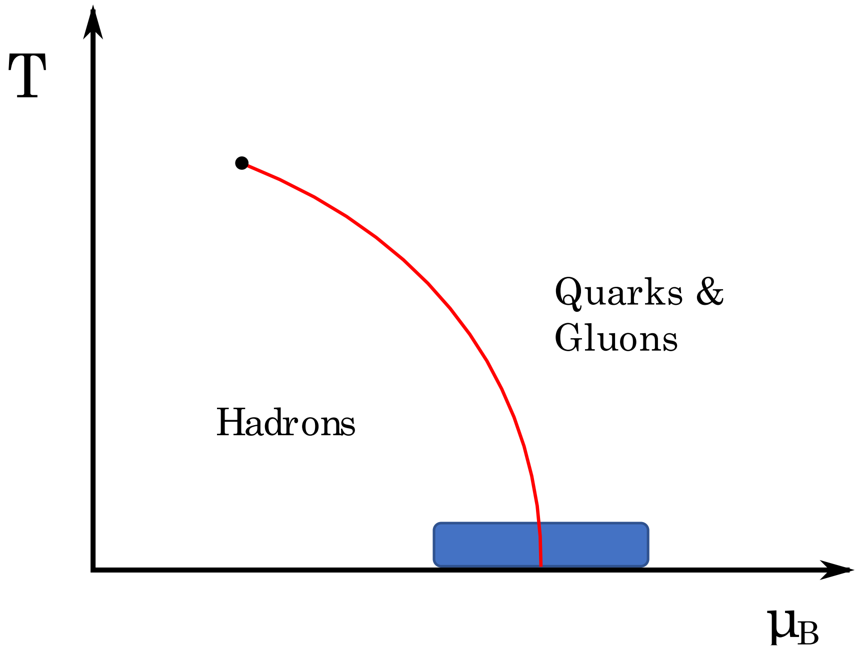
Savvas Pitsinigkos

Strong and Electroweak Matter 2024

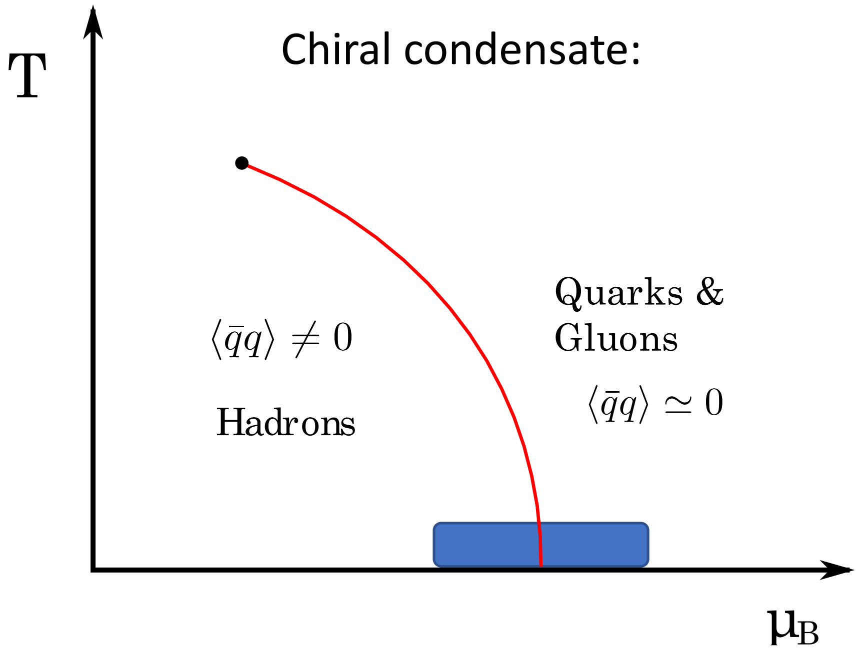


S. Pitsinigkos, A. Schmitt, Phys. Rev. D 109, 014024 (2024)

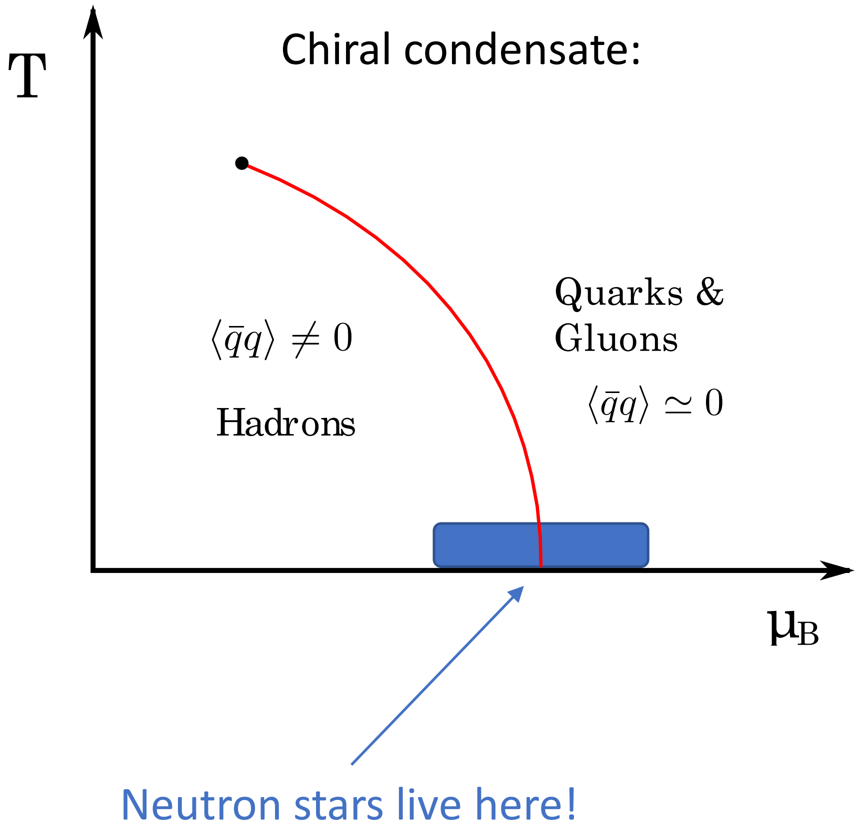
Motivation



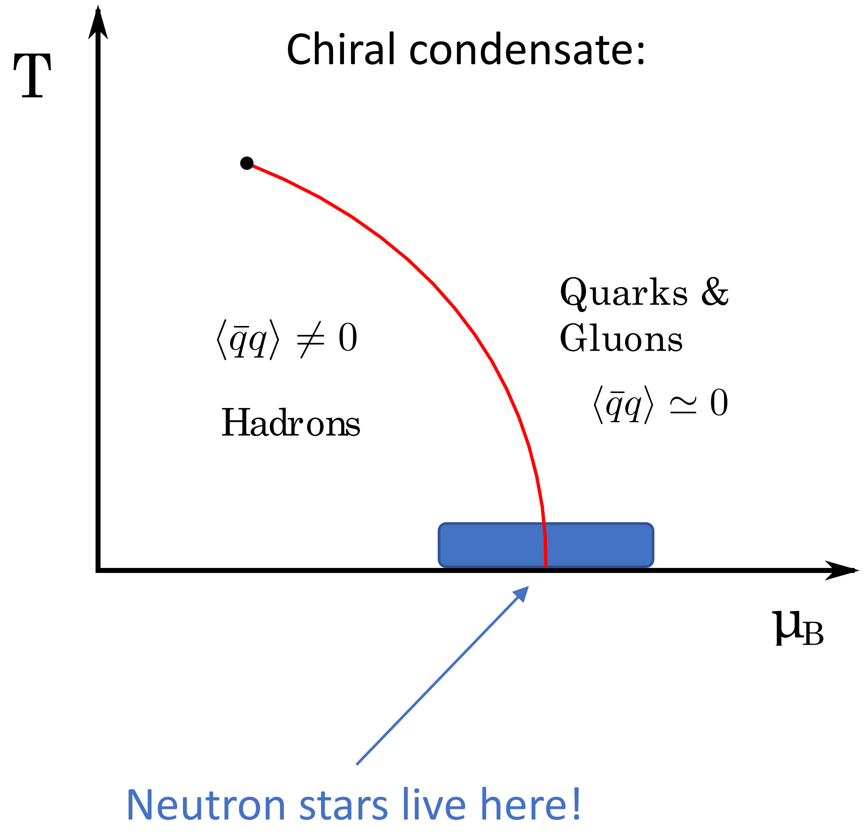
Motivation


T
Chiral condensate:
$\langle \bar{q}q \rangle \neq 0$
Hadrons
Quarks &
Gluons
$\langle \bar{q}q \rangle \simeq 0$
μ_B

Motivation


T
Chiral condensate:
$\langle \bar{q}q \rangle \neq 0$
Hadrons
Quarks &
Gluons
$\langle \bar{q}q \rangle \simeq 0$
μ_B
Neutron stars live here!

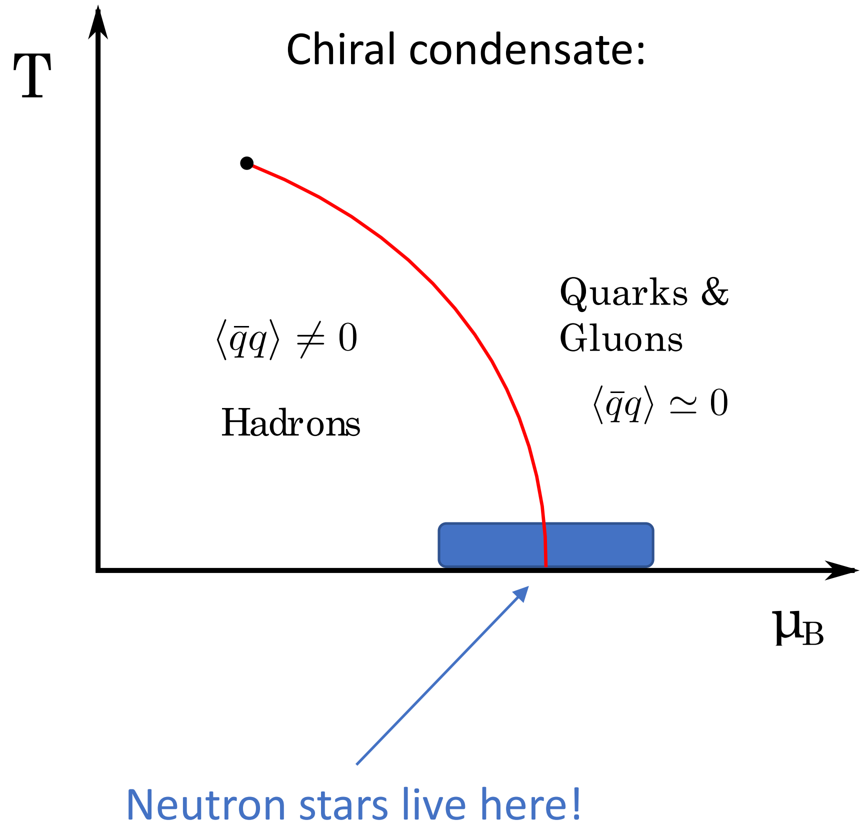
Motivation



What is the
phase structure?

What if the ground
state is not isotropic?

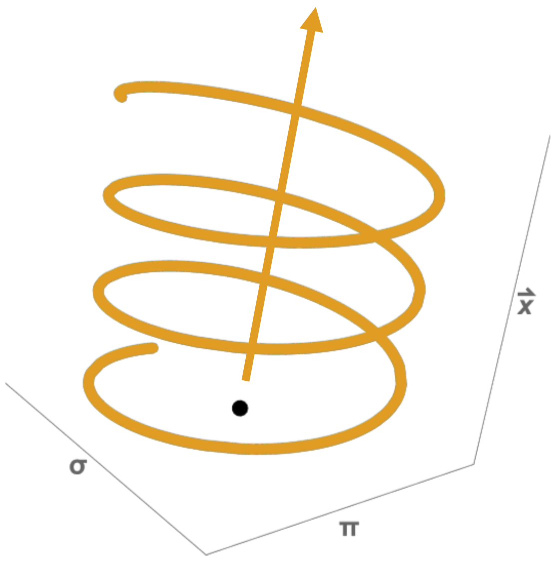
Motivation



What is the phase structure?

What if the ground state is not isotropic?

Anisotropic chiral condensate



$$\langle \sigma \rangle = \phi \cos(\vec{q} \cdot \vec{x}), \; \langle \pi_0 \rangle = \phi \sin(\vec{q} \cdot \vec{x})$$

Chiral Density Wave (CDW)

A. Heinz, F. Giacosa, and D. H. Rischke, Nucl. Phys. A 933, 34 (2015) + referenfces therein
S. Carignano, M. Buballa, PRD, 101(1), 014026 (2020).

Model and novelties

Used to study the chiral phase transition

E. Fraga, R. da Mata, **SP**, A. Schmitt, Phys. Rev. D 106 7, 074018 (2022)
Andreas Schmitt, Phys. Rev. D 101, 074007
Eduardo S. Fraga, Maurício Hippert, and Andreas Schmitt Phys. Rev. D 99, 014046

Nucleon-meson model:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu + \gamma^0\mu)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{4}\mathrm{Tr}[\partial_\mu\pi\partial^\mu\pi] - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{m_\omega^2}{2}\omega_\mu\omega^\mu + \frac{d}{4}(\omega_\mu\omega^\mu)^2 - \mathcal{U}(\sigma,\pi)$$
$$-\bar{\psi}\left[g_\sigma(\sigma + i\gamma^5\pi) + g_\omega\gamma^\mu\omega_\mu\right]\psi$$

$$\mathcal{U}(\sigma,\pi) = \sum_{n=1}^{4}\frac{a_n}{n!}\frac{(\sigma^2 + \pi_a\pi^a - f_\pi^2)^n}{2^n} - \epsilon(\sigma - f_\pi)$$

Model and novelties

Used to study the chiral phase transition

E. Fraga, R. da Mata, **SP**, A. Schmitt, Phys. Rev. D 106 7, 074018 (2022)
Andreas Schmitt, Phys. Rev. D 101, 074007
Eduardo S. Fraga, Maurício Hippert, and Andreas Schmitt Phys. Rev. D 99, 014046

Nucleon-meson model:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu + \gamma^0\mu)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{4}\mathrm{Tr}[\partial_\mu\pi\partial^\mu\pi] - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{m_\omega^2}{2}\omega_\mu\omega^\mu + \frac{d}{4}(\omega_\mu\omega^\mu)^2 - \mathcal{U}(\sigma,\pi)$$
$$-\bar{\psi}\left[g_\sigma(\sigma + i\gamma^5\pi) + g_\omega\gamma^\mu\omega_\mu\right]\psi$$

$$\mathcal{U}(\sigma,\pi) = \sum_{n=1}^{4}\frac{a_n}{n!}\frac{(\sigma^2 + \pi_a\pi^a - f_\pi^2)^n}{2^n} - \epsilon(\sigma - f_\pi)$$

No explicit mass terms for nucleons $M = g_\sigma\phi$,
Chiral symmetry "restoration" possible

Model and novelties

Used to study the chiral phase transition

E. Fraga, R. da Mata, **SP**, A. Schmitt, Phys. Rev. D 106 7, 074018 (2022)
Andreas Schmitt, Phys. Rev. D 101, 074007
Eduardo S. Fraga, Maurício Hippert, and Andreas Schmitt Phys. Rev. D 99, 014046

Nucleon-meson model:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu + \gamma^0\mu)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{4}\mathrm{Tr}[\partial_\mu\pi\partial^\mu\pi] - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{m_\omega^2}{2}\omega_\mu\omega^\mu + \frac{d}{4}(\omega_\mu\omega^\mu)^2 - \mathcal{U}(\sigma,\pi)$$
$$-\bar{\psi}\left[g_\sigma(\sigma + i\gamma^5\pi) + g_\omega\gamma^\mu\omega_\mu\right]\psi$$

$$\mathcal{U}(\sigma,\pi) = \sum_{n=1}^{4}\frac{a_n}{n!}\frac{(\sigma^2+\pi_a\pi^a - f_\pi^2)^n}{2^n} - \epsilon(\sigma - f_\pi)$$

No explicit mass terms for nucleons $M = g_\sigma\phi$,
Chiral symmetry “restoration” possible

- Mean field free energy and $T = 0$
- Impose isospin symmetry $n_p = n_n$

Model and novelties

Used to study the chiral phase transition

E. Fraga, R. da Mata, **SP**, A. Schmitt, Phys. Rev. D 106 7, 074018 (2022)
Andreas Schmitt, Phys. Rev. D 101, 074007
Eduardo S. Fraga, Maurício Hippert, and Andreas Schmitt Phys. Rev. D 99, 014046

Nucleon-meson model:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu + \gamma^0\mu)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{4}\mathrm{Tr}[\partial_\mu\pi\partial^\mu\pi] - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{m_\omega^2}{2}\omega_\mu\omega^\mu + \frac{d}{4}(\omega_\mu\omega^\mu)^2 - \mathcal{U}(\sigma,\pi)$$
$$-\bar{\psi}\left[g_\sigma(\sigma + i\gamma^5\pi) + g_\omega\gamma^\mu\omega_\mu\right]\psi$$

$$\mathcal{U}(\sigma,\pi) = \sum_{n=1}^{4}\frac{a_n}{n!}\frac{(\sigma^2 + \pi_a\pi^a - f_\pi^2)^n}{2^n} - \epsilon(\sigma - f_\pi)$$

No explicit mass terms for nucleons $M = g_\sigma\phi$,
Chiral symmetry "restoration" possible

- Mean field free energy and $T = 0$
- Impose isospin symmetry $n_p = n_n$

Similar in spirit approaches:

- Inhomogeneous phases extensively studied in quark models

S. Carignano, M. Buballa, B.J. Schaefer, Phys. Rev. D 90, 014033 (2014)

- Nucleon models ignore Dirac sea

A. Heinz, F. Giacosa, and D. H. Rischke, Nucl. Phys. A 933, 34 (2015)

Dirac sea and renormalization

Dirac sea and renormalization

Why?

Dirac sea and renormalization

Why?

Significant contribution with qualitative effects for the chiral transition

Brandes L., Kaiser N. & Weise W. Eur. Phys. J. A 57, 243 (2021)

1^{st} order → Crossover

Dirac sea and renormalization

Why?

Baryonic vacuum contribution:

$$P_{\mathrm{vac}}(\phi, q) = \sum_{n,p} \int \frac{d^3\vec{k}}{(2\pi)^3} E_k(\phi, q)$$

divergent!

Significant contribution with qualitative effects for the chiral transition

Brandes L., Kaiser N. & Weise W. Eur. Phys. J. A 57, 243 (2021)

1st order → Crossover

Dirac sea and renormalization

Why?

Baryonic vacuum contribution:

$$P_{\text{vac}}(\phi, q) = \sum_{n,p} \int \frac{d^3\vec{k}}{(2\pi)^3} E_k(\phi, q)$$

divergent!

Proper time regularization Λ + renormalization at scale ℓ

Significant contribution with qualitative effects for the chiral transition

Brandes L., Kaiser N. & Weise W. Eur. Phys. J. A 57, 243 (2021)

1^{st} order → Crossover

Dirac sea and renormalization

Why?

Baryonic vacuum contribution:

$$P_{\text{vac}}(\phi, q) = \sum_{n,p} \int \frac{d^3\vec{k}}{(2\pi)^3} E_k(\phi, q)$$

divergent!

Significant contribution with qualitative effects for the chiral transition

Brandes L., Kaiser N. & Weise W. Eur. Phys. J. A 57, 243 (2021)

1st order → Crossover

Proper time regularization Λ + renormalization at scale ℓ

Select $\ell = \sqrt{m_N^2 + (2q)^2}$

Eduardo S. Fraga, Letícia F. Palhares, and Tulio E. Restrepo
Phys. Rev. D 108, 034026 (2023)
Aleksi Kurkela, Paul Romatschke, and Aleksi Vuorinen
Phys. Rev. D 81, 105021 (2010)

Dirac sea and renormalization

Why?

Baryonic vacuum contribution:

$$P_{\text{vac}}(\phi, q) = \sum_{n,p} \int \frac{d^3\vec{k}}{(2\pi)^3} E_k(\phi, q)$$

divergent!

Proper time regularization Λ + renormalization at scale ℓ

So that free energy is bounded for large q:

$$\Omega = g_\sigma^2 \frac{q^2\phi^2}{2\pi^2}\left(2 + \frac{4\pi^2}{g_\sigma^2} - \ln\frac{4q^2}{\ell^2}\right) + \mathcal{O}(q^0)$$

Significant contribution with qualitative effects for the chiral transition

Brandes L., Kaiser N. & Weise W. Eur. Phys. J. A 57, 243 (2021)

1^{st} order → Crossover

Select $\ell = \sqrt{m_N^2 + (2q)^2}$

Eduardo S. Fraga, Letícia F. Palhares, and Tulio E. Restrepo Phys. Rev. D 108, 034026 (2023)

Aleksi Kurkela, Paul Romatschke, and Aleksi Vuorinen Phys. Rev. D 81, 105021 (2010)

Fixing the model parameters

Free parameters ↔ properties of *symmetric nuclear matter at saturation*:

- Binding energy: $E_B = -16.3\ \mathrm{MeV}$
- Density: $n_0 = 0.15\ \mathrm{fm}^{-3}$
- Incompressibility: $K \approx (200 - 300)\ \mathrm{MeV}$
- Symmetry energy: $S \approx (30 - 34)\ \mathrm{MeV}$
- Slope of symmetry energy: $L \approx (40 - 140)\ \mathrm{MeV}$ (PREX), Phys. Rev. Lett. 126, 172502 (2021)
- Effective nucleon mass: $M_0 \approx (0.7 - 0.8) m_N$

Fixing the model parameters

Free parameters ↔ properties of *symmetric nuclear matter at saturation*:

- Binding energy: $E_B = -16.3\ \mathrm{MeV}$
- Density: $n_0 = 0.15\ \mathrm{fm}^{-3}$
- Incompressibility: $K \approx (200 - 300)\ \mathrm{MeV}$
- Symmetry energy: $S \approx (30 - 34)\ \mathrm{MeV}$
- Slope of symmetry energy: $L \approx (40 - 140)\ \mathrm{MeV}$ (PREX), Phys. Rev. Lett. 126, 172502 (2021)
- Effective nucleon mass: $M_0 \approx (0.7 - 0.8)m_N$

We mainly explore the
M_0, L
parameter space

Fixing the model parameters

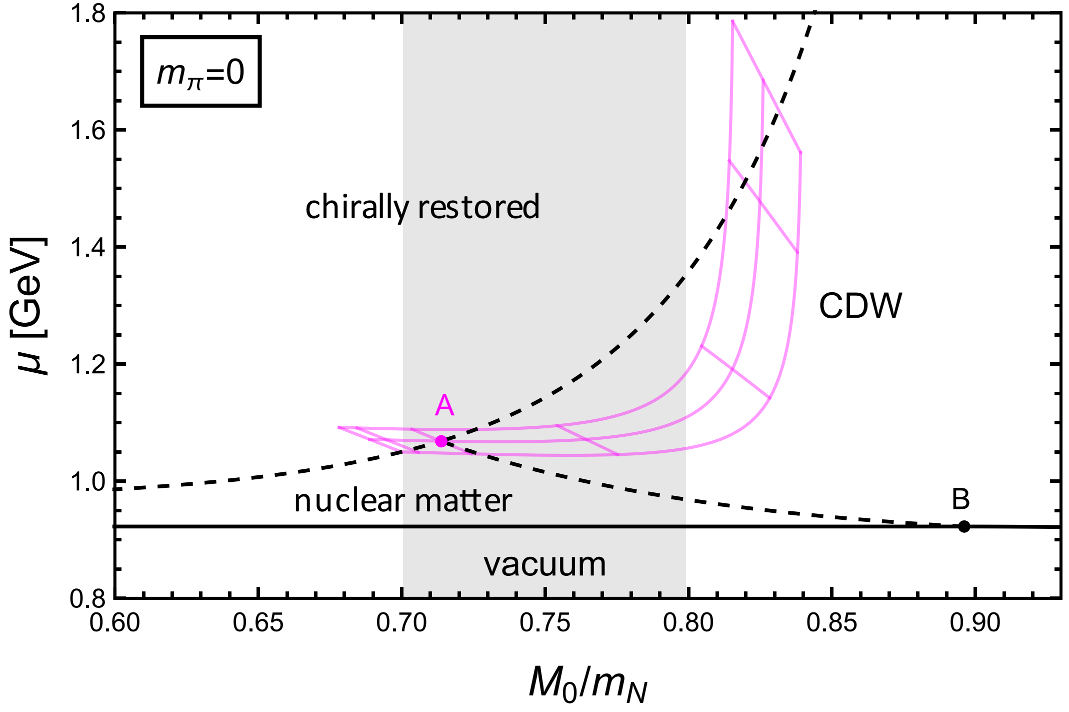
Free parameters ↔ properties of *symmetric nuclear matter at saturation*:

- Binding energy: $E_B = -16.3\ \mathrm{MeV}$
- Density: $n_0 = 0.15\ \mathrm{fm}^{-3}$
- Incompressibility: $K \approx (200 - 300)\ \mathrm{MeV}$
- Symmetry energy: $S \approx (30 - 34)\ \mathrm{MeV}$
- Slope of symmetry energy: $L \approx (40 - 140)\ \mathrm{MeV}$ (PREX), Phys. Rev. Lett. 126, 172502 (2021)
- Effective nucleon mass: $M_0 \approx (0.7 - 0.8) m_N$

We mainly explore the
M_0, L
parameter space

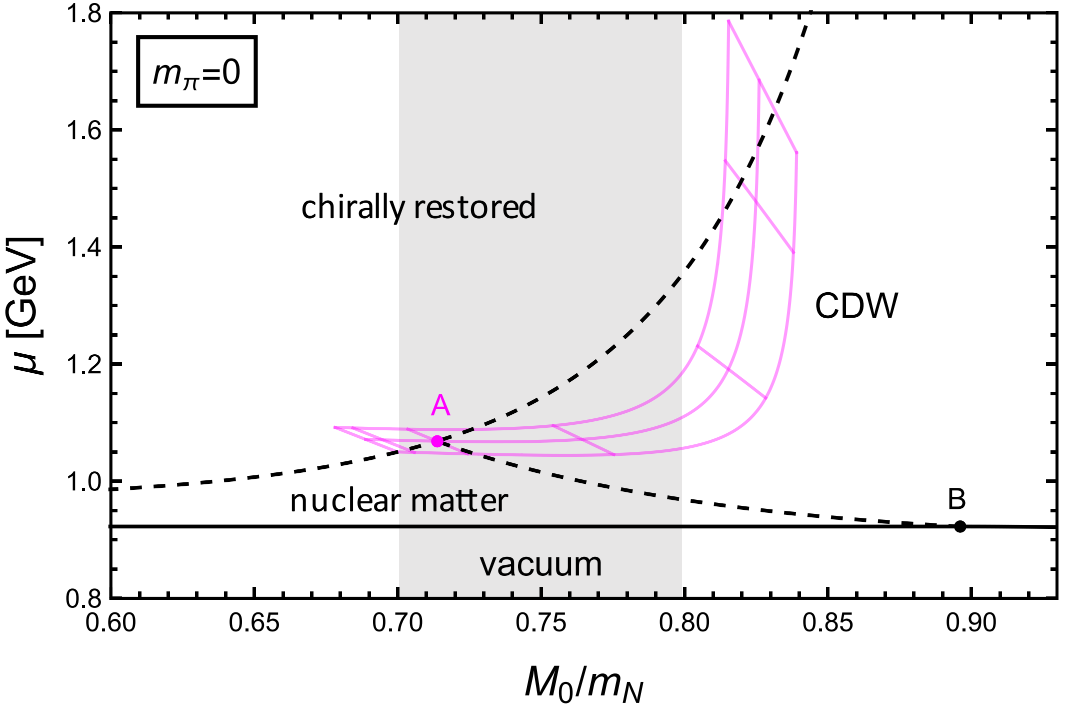
Direct relation to nuclear matter phenomenology!

Results - CDW

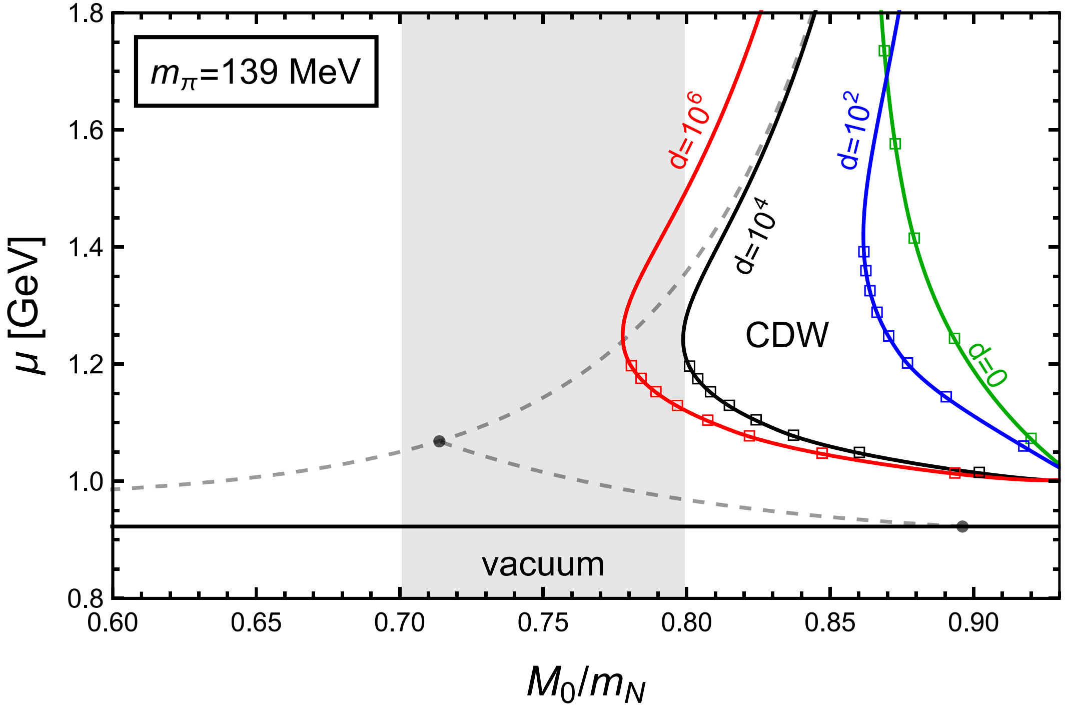


CDW replaces the chiral phase transition

Results - CDW

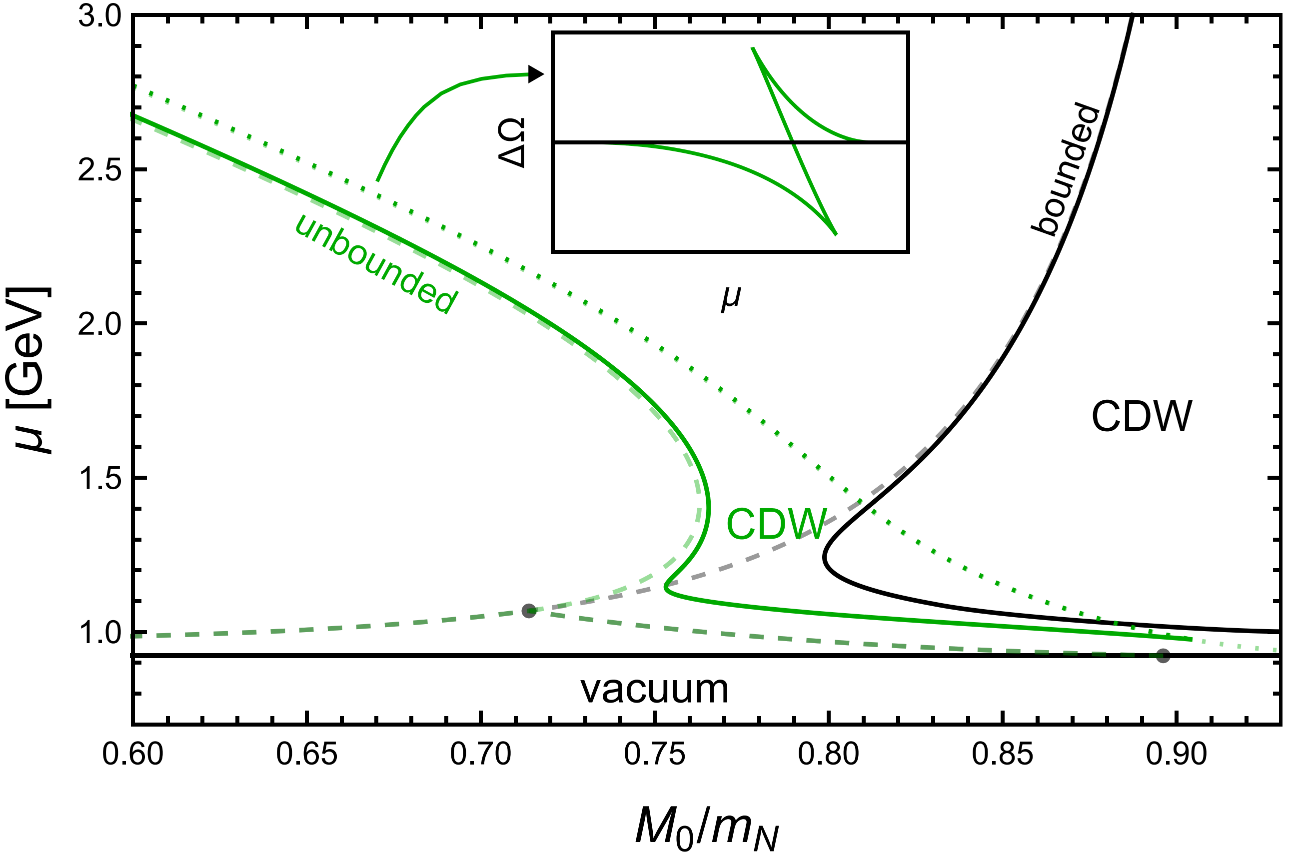


CDW replaces the chiral phase transition



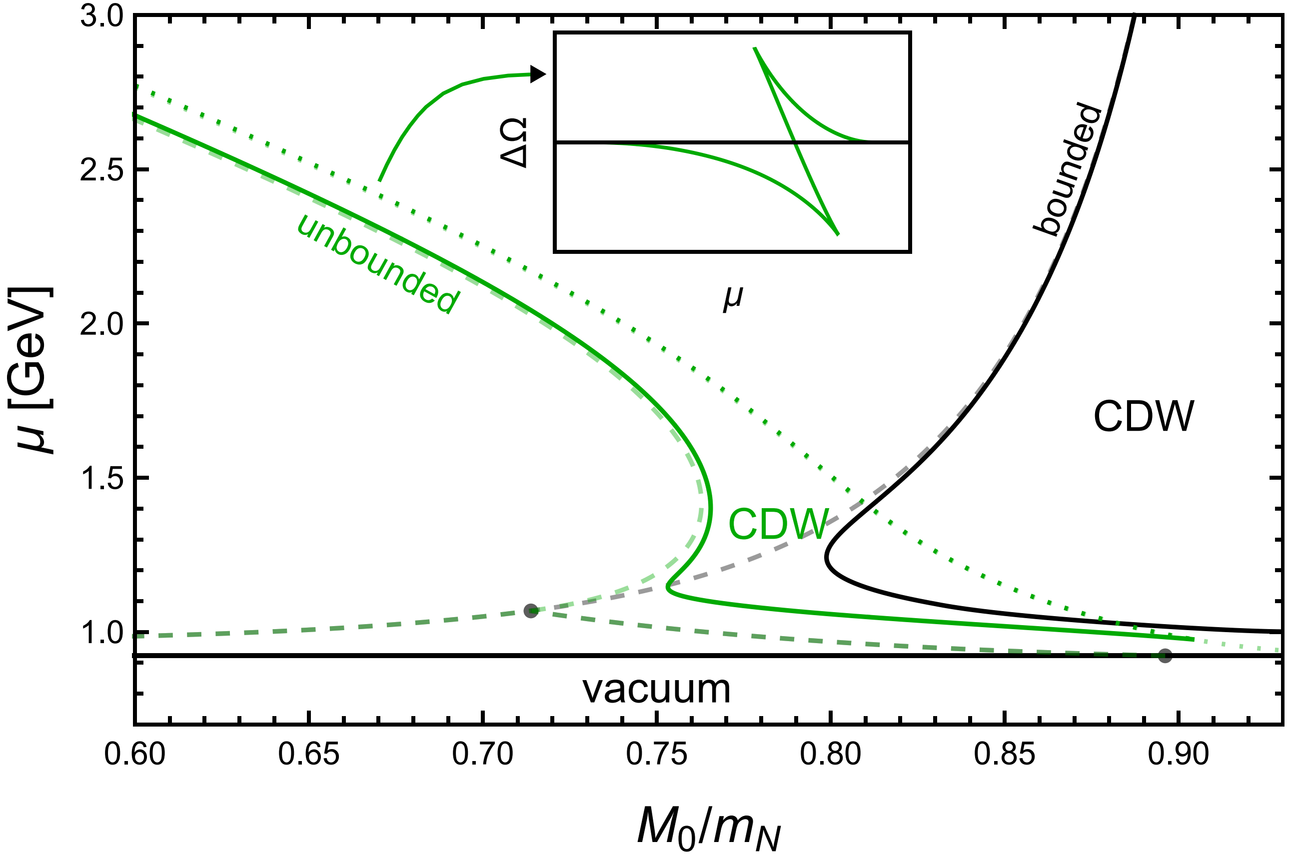
Explicit chiral symmetry breaking shifts the CDW

Results - CDW

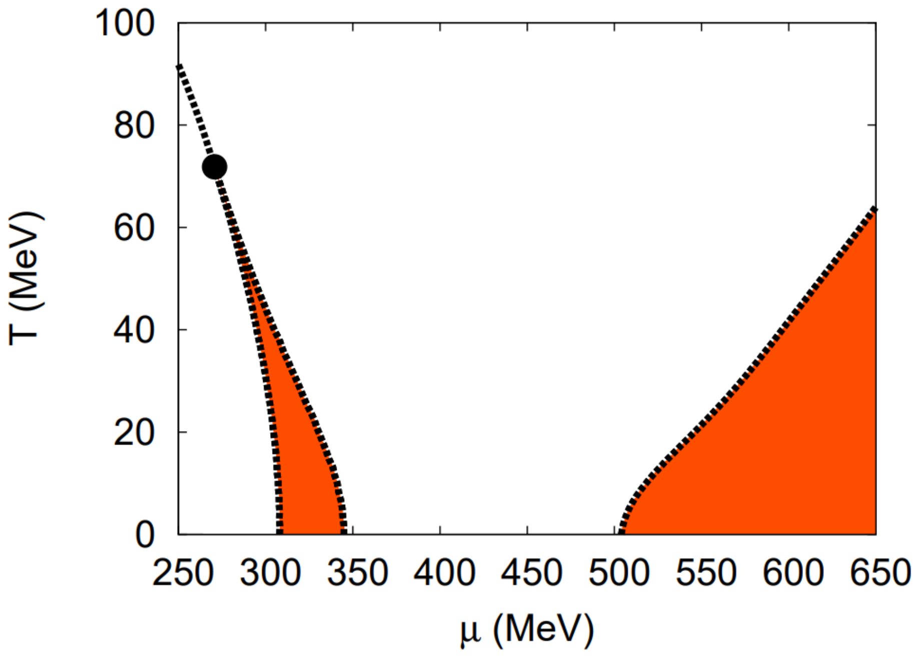


Without the q-dependent renormalization scale the unphysical island appears

Results - CDW



Without the q-dependent renormalization scale the unphysical island appears



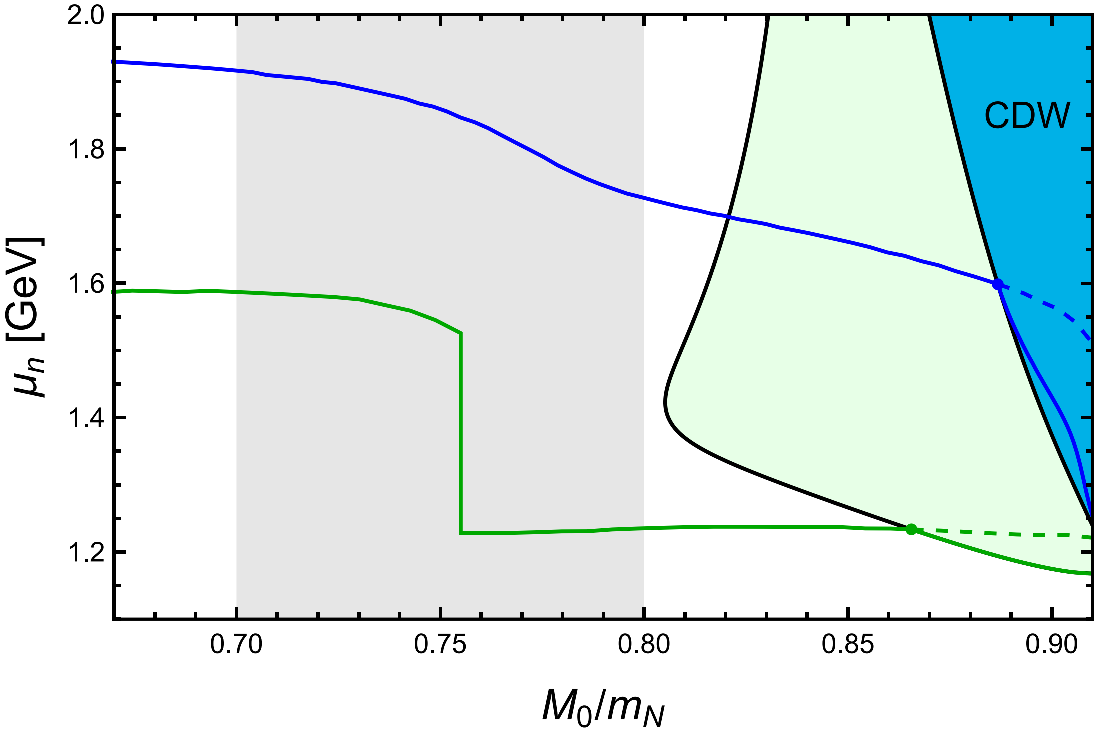
S. Carignano, M. Buballa, Acta Phys. Pol. B Proc. Suppl. 5, 641 (2012)

CDW in Neutron Stars?

Preliminary

Poster: “Chiral Density Wave under neutron star conditions”

O. Papadopoulos and A. Schmitt – Work in progress

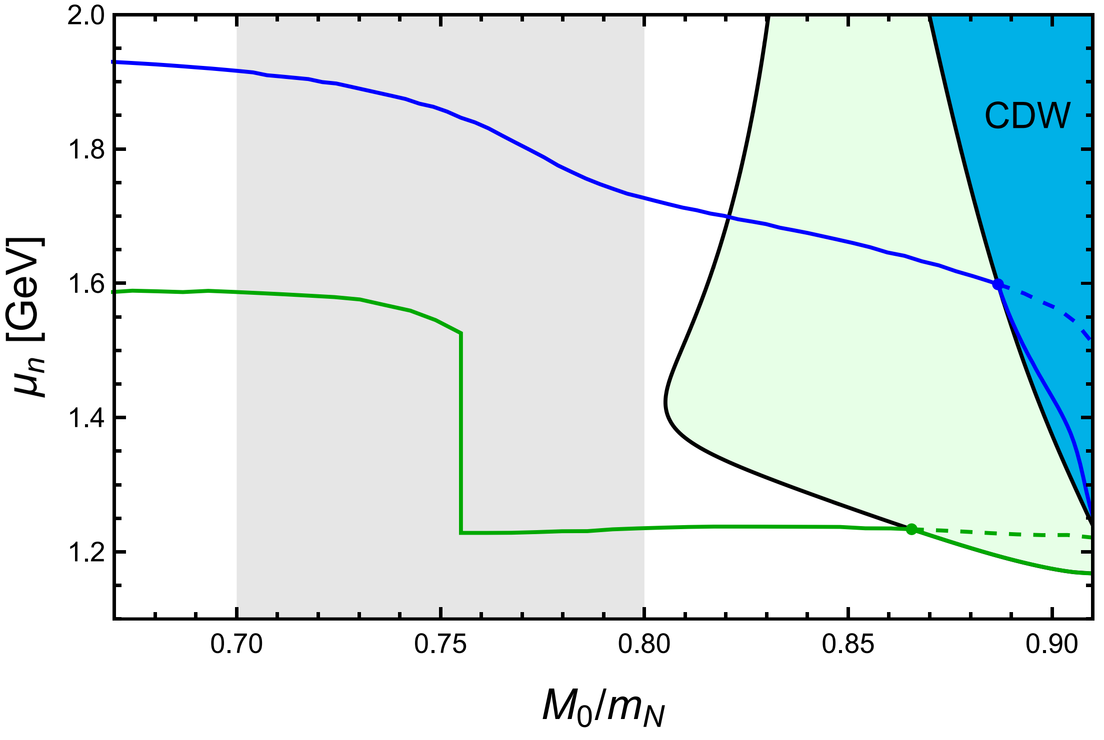


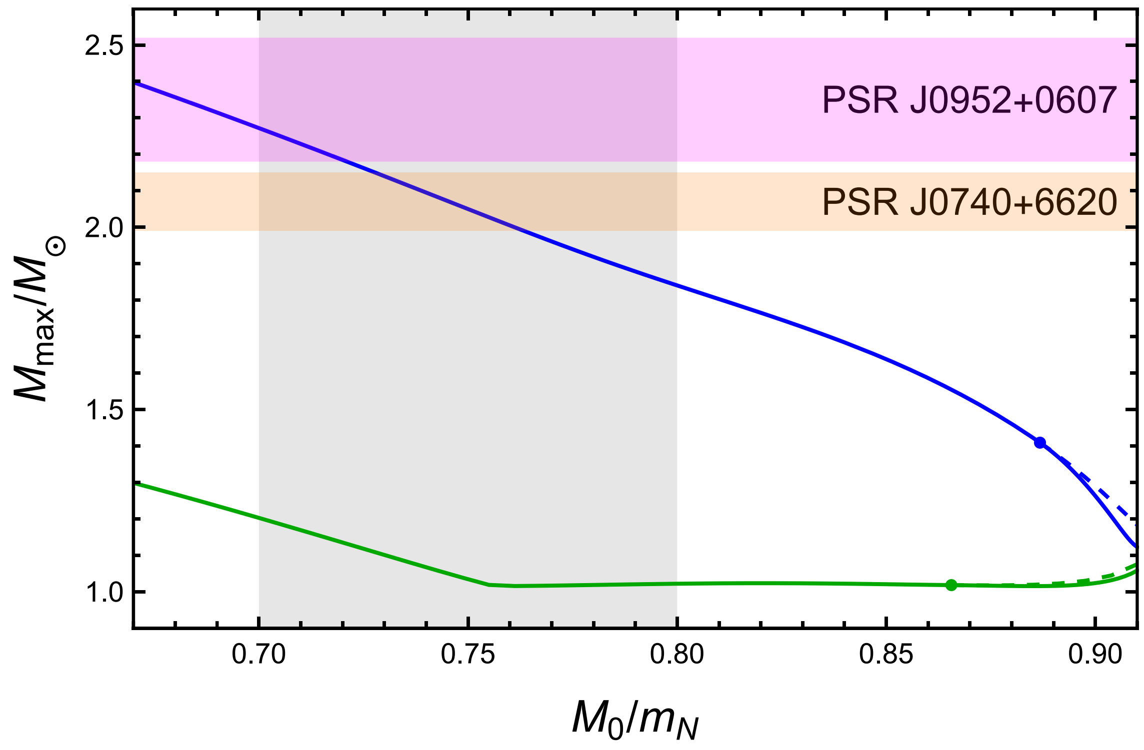
CDW in Neutron Stars?

Preliminary

Poster: “Chiral Density Wave under neutron star conditions”

O. Papadopoulos and A. Schmitt – Work in progress



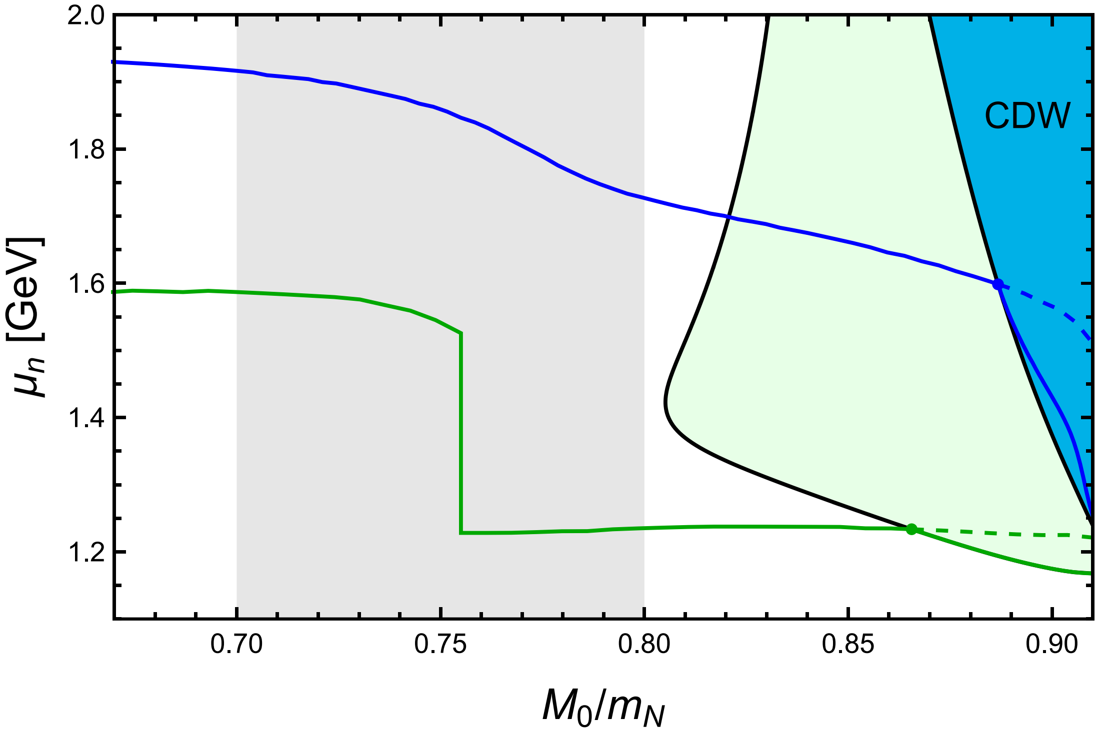


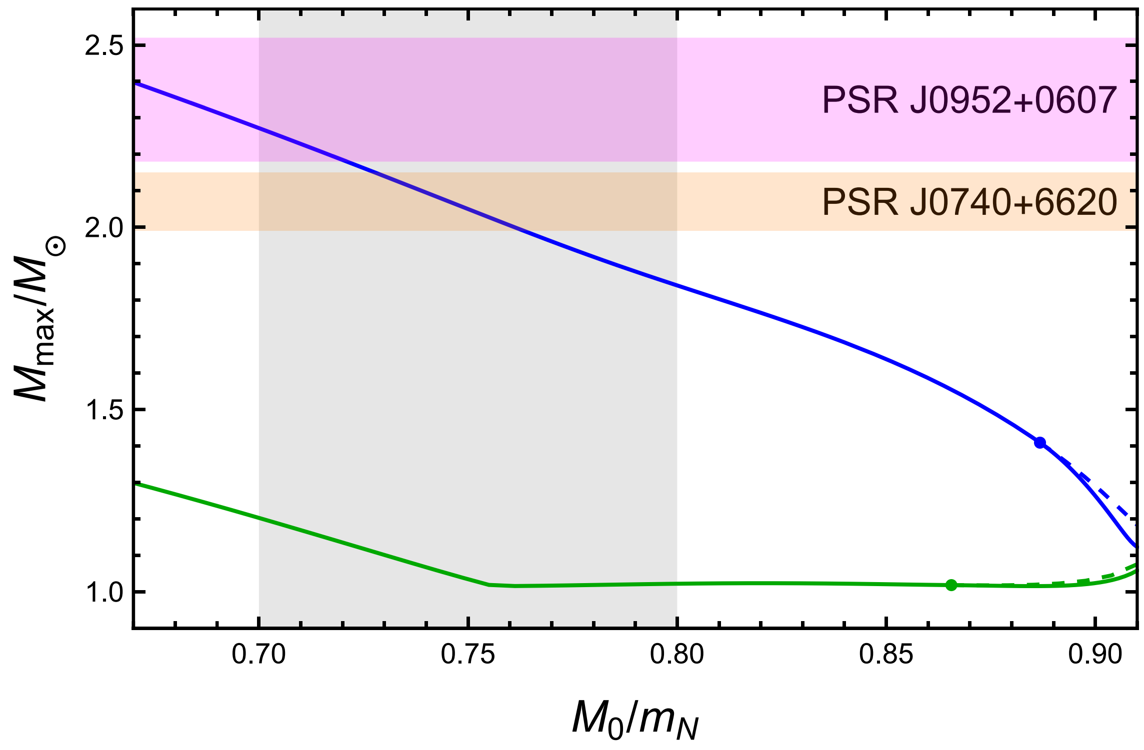
CDW in Neutron Stars?

Poster: “Chiral Density Wave under neutron star conditions”

Preliminary

O. Papadopoulos and A. Schmitt – Work in progress





Existence of heavy compact stars excludes CDW

Summary and Outlook

Summary and Outlook

- Setup a nucleonic model for the CDW

Summary and Outlook

- Setup a nucleonic model for the CDW
- Included the Dirac sea

Summary and Outlook

- Setup a nucleonic model for the CDW
- Included the Dirac sea
- Used a renormalization scheme to cure previously observed unphysical behavior

Summary and Outlook

- Setup a nucleonic model for the CDW
- Included the Dirac sea
- Used a renormalization scheme to cure previously observed unphysical behavior
- Mapped the region in the parameter space where CDW is preferred

Summary and Outlook

- Setup a nucleonic model for the CDW
- Included the Dirac sea
- Used a renormalization scheme to cure previously observed unphysical behavior
- Mapped the region in the parameter space where CDW is preferred

Outlook

- Beta equilibrium, charge neutrality — O. Papadopoulos and A. Schmitt – In preparation
- Alternative models to tackle model dependent effects — A. Heinz, F. Giacosa, D. H. Rischke, Nucl. Phys. A, 933, 34-42, (2015)
- Transport properties may be significantly modified — With implications for: M. Alford, A. Harutyunyan, A. Sedrakian, Phys. Rev. D 108, 083019 (2023)

Summary and Outlook

- Setup a nucleonic model for the CDW
- Included the Dirac sea
- Used a renormalization scheme to cure previously observed unphysical behavior
- Mapped the region in the parameter space where CDW is preferred

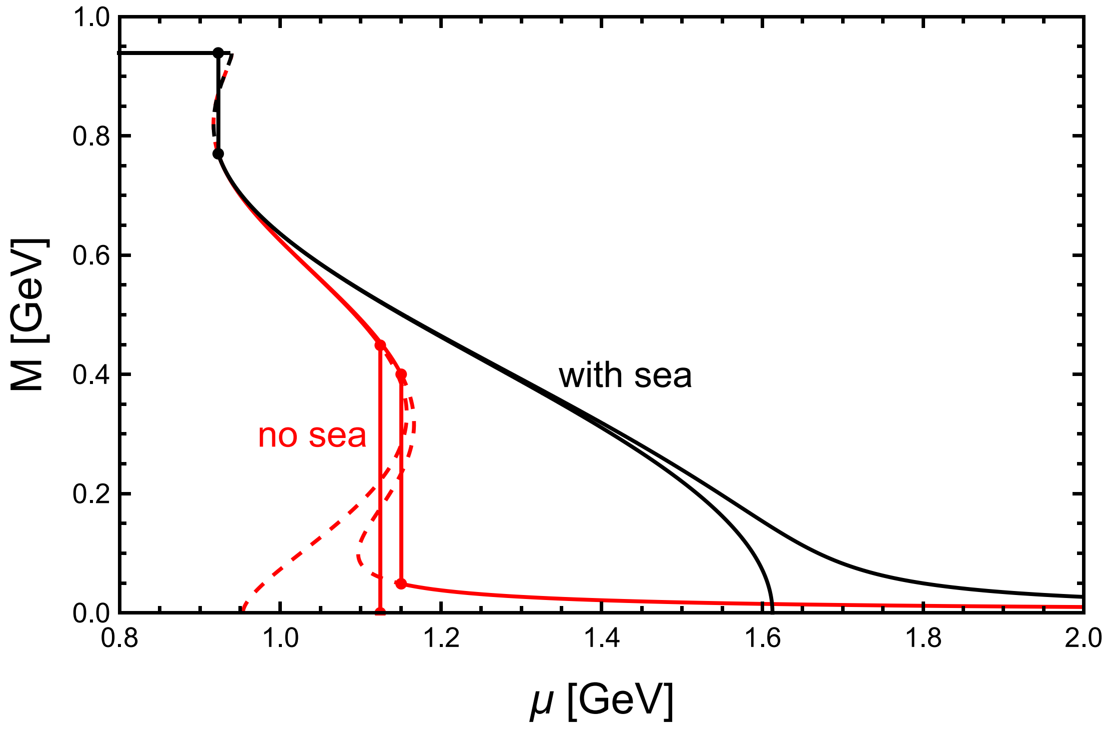
Outlook

- Beta equilibrium, charge neutrality — O. Papadopoulos and A. Schmitt – In preparation
- Alternative models to tackle model dependent effects — A. Heinz, F. Giacosa, D. H. Rischke, Nucl. Phys. A, 933, 34-42, (2015)
- Transport properties may be significantly modified — With implications for: M. Alford, A. Harutyunyan, A. Sedrakian, Phys. Rev. D 108, 083019 (2023)

Thank you!

Extras

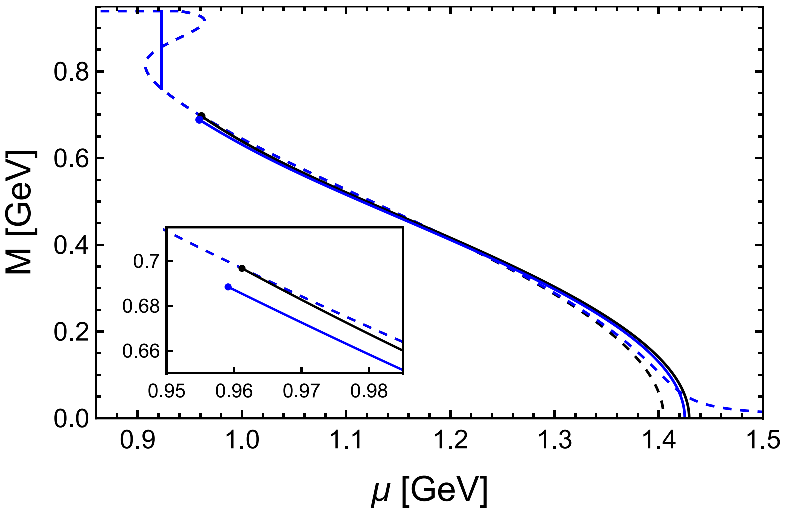
Results - Isotropic



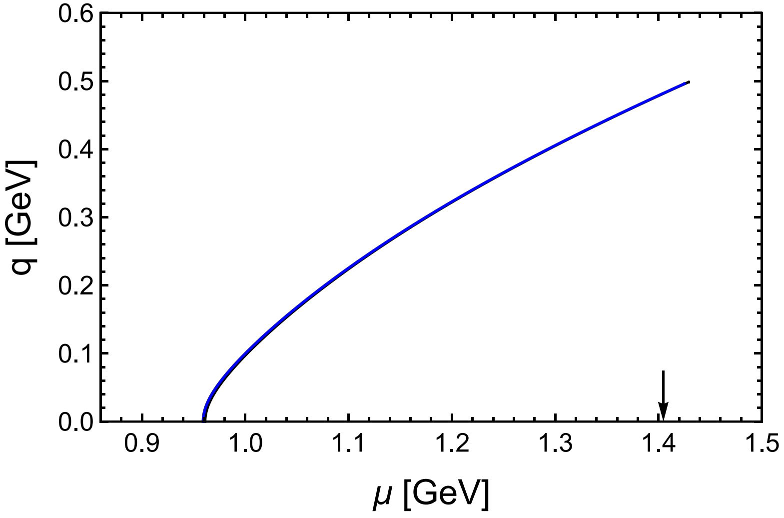
The Dirac sea's effect is important even for homogeneous phases

Brandes L., Kaiser N. & Weise W. Eur. Phys. J. A 57, 243 (2021)

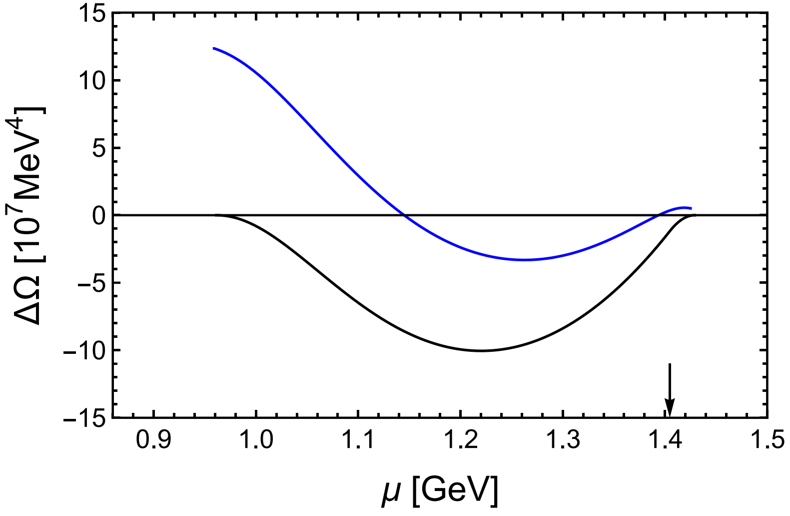
Results - CDW



CDW appears despite the crossover



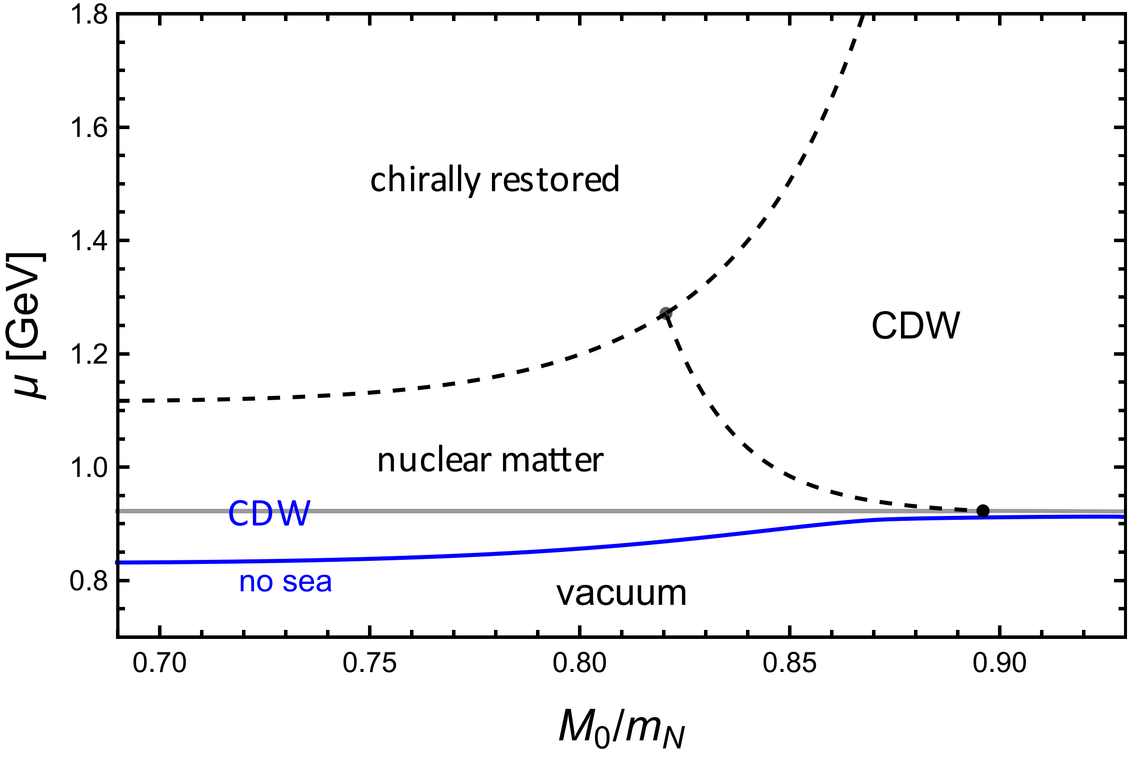
Small effect of the pion mass



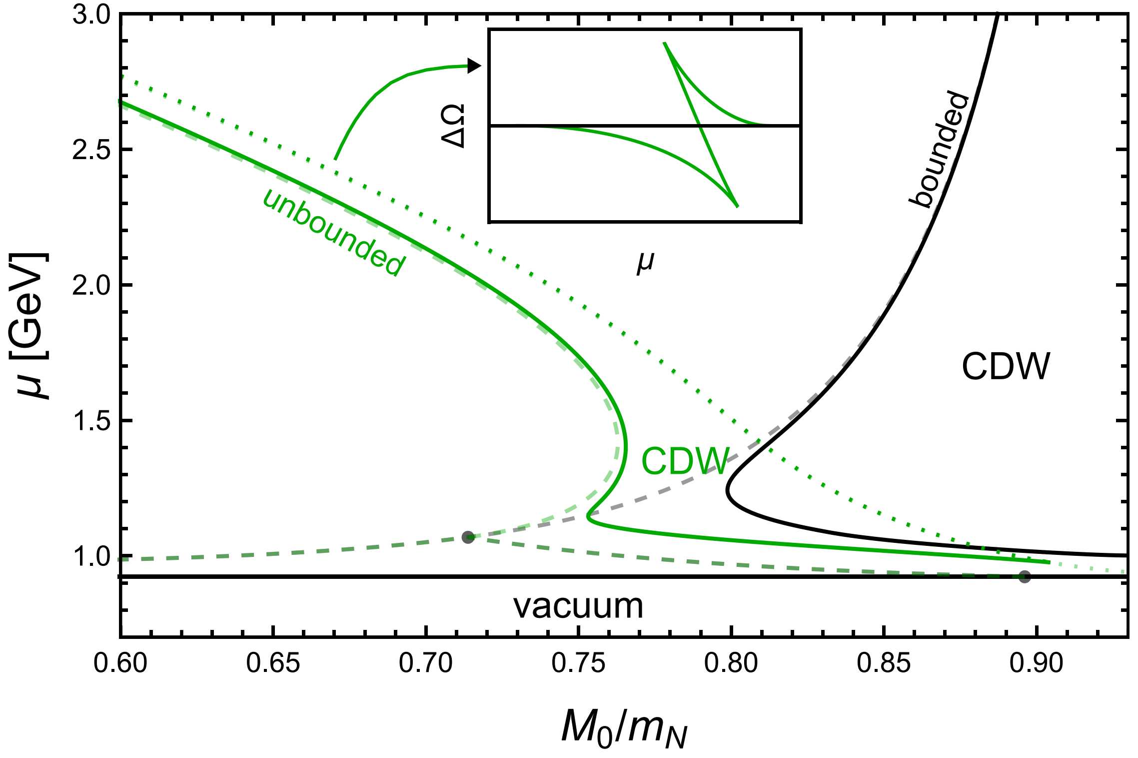
CDW preferred when $\Delta\Omega < 0$

Chiral limit – Physical pion mass

Results - CDW



Without the sea the CDW is always preferred



Without the q-dependent renormalization scale the unphysical island appears