A03
Inhomogeneous phases at high density

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GOETHE UNIVERSITÄT FRANKFURT AM MAIN

CRC-TR 211 Strong-interaction matter under extreme conditions

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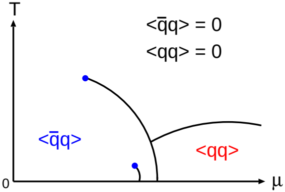
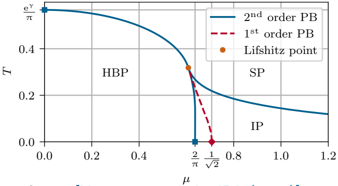
Background: Inhomogeneous phases

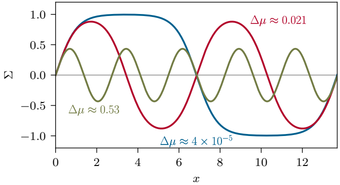
- QCD phase diagram (standard picture):



Usual assumption:
condensates (= order parameters)
spatially constant

- Mean-field model calculations (Gross-Neveu, NJL, QM model):
Phases with non-uniform condensates favored in certain regions





from: [Koenigstein et al., JPA (2022)], see also: [Thies, Urlichs, PRD (2003)]

Main questions

Research goals continued from the 1st funding period:

- What is the energetically preferred spatial shape of the order parameter?
- Inhomogeneous phases found so far: mostly mean-field results.
  $\rightarrow$ Are they stable under thermal and quantum fluctuations?
- How do the inhomogeneous phases depend on the parameters (number of spacetime dimensions, quark masses, isospin chemical potential, strangeness chemical potential)?

New direction in the 2nd funding period:

- How do inhomogeneous chiral phases compete with homogeneous or inhomogeneous color superconducting phases?

Approach

Inhomogeneous chiral phases expected at large μ and small T
$\rightarrow$ not accessible by lattice QCD
$\rightarrow$ use QCD-inspired models to investigate these questions

(But: recent efforts to employ QCD Dyson-Schwinger equations [not part of the CRC proposal])

Methods:

- Lattice Field Theory (LFT)
  - Mean field: minimizations of the discretized effective action
  - Lattice Monte Carlo simulations
- Continuum methods, in particular the Functional Renormalization Group (FRG)

Both methods complement each other and their combination allows to investigate systematic errors:

- Discretization errors and finite volume corrections (LFT)
- Truncation effects (FRG)

Q1: Preferred spatial shape of the order parameter

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example: Gross-Neveu model in D dimensions

- Mean-field effective potential:

$$S_{\text{eff}}[\sigma] = N_f \left[\frac{1}{2\lambda} \int d^D x \, \sigma^2 - \ln \text{Det}(\partial\!\!\!/ + \gamma_0 \mu + \sigma) \right]$$

  - $\sigma = -\frac{\lambda}{N_f} \langle \bar{\psi} \psi \rangle$ chiral order parameter
  - λ : coupling constant

- Preferred state: minimize S_{eff} w.r.t. σ
  - restriction to homogeneous matter ($\sigma = \text{const}$) $\rightarrow$ easy
  - inhomogeneous matter ($\sigma(\vec{x})$) $\rightarrow$ very difficult

- Approaches so far:
  - stability analysis: Is the preferred *homogeneous* solution stable against small inhomogeneous fluctuations?
    $\rightarrow$ existence of an inhom. phase, but no information about the shape
  - restricted ansatz functions for $\sigma(\vec{x})$
  - LFT analysis [Wagner, PRD (2007); Heinz Giacosa, Wagner, Rischke, PRD (2017)]
    was restricted to 1D spatial modulations $\rightarrow$ extend to 3D

Q2: Stability under fluctuations

Inhomogeneous phases found so far: mostly mean-field results

- Do they survive beyond mean-field?
- Or are they washed out by bosonic fluctuations (Goldstone modes)?

FRG approach:

- Wetterich equation: $\partial_k \Gamma_k(\phi) = \frac{1}{2}\mathrm{STr}\Big\{ \big[\Gamma_k^{(2)}[\phi] + R_k\big]^{-1} \partial_k R_k \Big\}$
  - Γ_k: "effective average action"
    - classical action at UV scale $k = \Lambda$ (starting point)
    - full quantum effective action at IR scale $k = 0$
  - $\Gamma_k^{(2)}$: 2nd functional derivative w.r.t. field

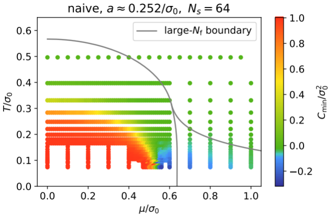
  exact, but hard (impossible) to solve, even for homogeneous phases

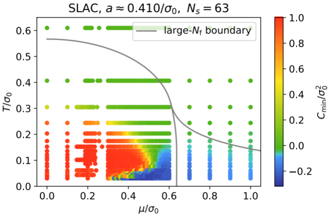
  $\rightarrow$ truncations needed
- Additional difficulty in inhomogeneous phases:
  $\Gamma_k^{(2)}$ non-diagonal in momentum space

  $\rightarrow$ main strategy: stability analysis of homogeneous solutions

LFT approach:

- Monte Carlo simulations of the path integral very expensive w.r.t. CPU time
- At the moment only results for the 1+1D GN model
  $\rightarrow$ Inhomogeneos region exists also in the presence of quantum fluctuations, but is significantly smaller

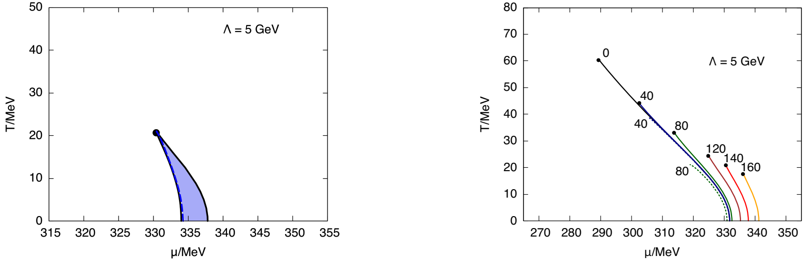




from: [Lenz, Pannullo, Wagner, Wellegehausen, Wipf, PRD (2020)]

Most earlier investigations: isospin-symmetric 2-flavor matter with $m_q = 0$

- Effect of non-zero m_q or strangeness degrees of freedom?



[Buballa, Carignano, Kurth, EPJST (2020)], see also: [Buballa, Carignano, PLB (2019)]

Mean-field studies in NJL and QM model:

- Inhomogeneous phase shrinks but survives for physical pion masses.
- Adding strange quarks: small effect [Carignano, Buballa, PRD (2020)]

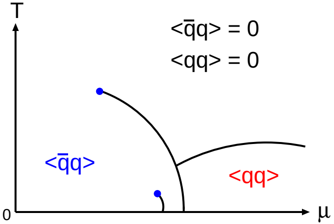
(How) does this change if fluctuations are included?

- Effect of isospin imbalance?

Q4: Competition with CSC

Color superconductivity: quark-quark pairing, order parameters $\langle q_i q_j \rangle$

- expected at large μ, low or moderate T
  $\rightarrow$ competes with chiral inhomogeneous phases



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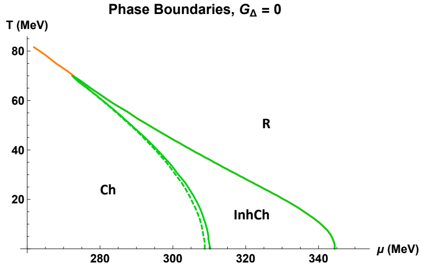

Mean-field NJL study:

[Lakaschus, Buballa, Rischke, PRD (2021)]

- possible coexistence phase
- CSC may replace inhomogeneous chiral phase completely
- strong dependence on qq coupling constant

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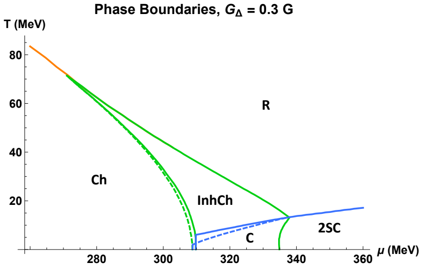

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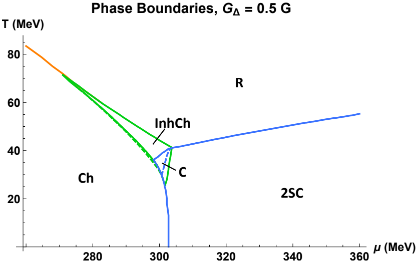

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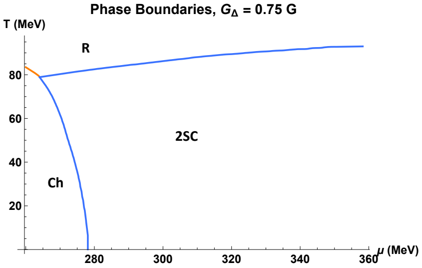

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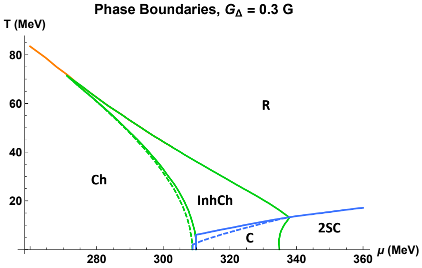

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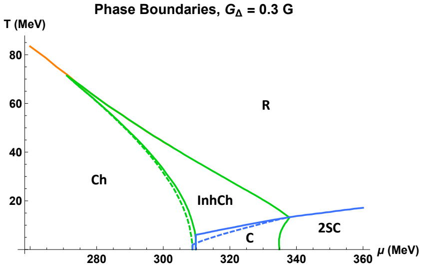

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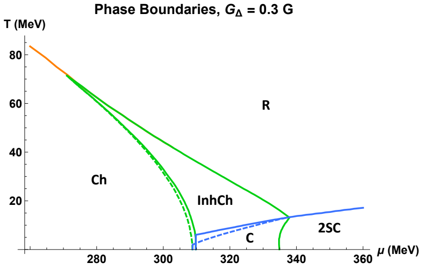

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- How does this change if fluctuations are included?
- Inhomogeneous CSC phases ("LOFF phases")
  - in isospin asymmetric matter?
  - in the coexistence phase?

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