A05

Topology of QCD

Dietrich Bödeker, Gergely Endrődi, Guy D. Moore

Bielefeld University (BU) and TU Darmstadt (TUDa)

Topology: Overview

CRC–TR 211

Starting point of CRC: $\mathcal{L}_{\rm QCD}$. Scalar, gauge-invariant, Dim ≤ 4:

$$\mathcal{L} = \text{(fermions)} + \frac{1}{4g^2} g^{\mu\alpha} g^{\nu\beta} G^A_{\mu\nu} G^A_{\alpha\beta} + \frac{\Theta}{64\pi^2} \epsilon^{\mu\nu\alpha\beta} G^A_{\mu\nu} G^A_{\alpha\beta} \qquad \text{(!)}$$

Θ-term violates P and T. Constrained to $|\Theta| < 10^{-10}$.
It's also a total derivative!

$$q(x) \equiv \frac{1}{64\pi^2} \epsilon^{\mu\nu\alpha\beta} G^A_{\mu\nu} G^A_{\alpha\beta}(x) = \partial_\mu K^\mu$$

$$K^\mu = \frac{1}{32\pi^2} \epsilon^{\mu\nu\alpha\beta} \left(G^A_\nu G^A_{\alpha\beta} - \frac{1}{3} f_{ABC} G^A_\nu G^B_\alpha G^C_\beta \right) ,$$

$$\int d^4x \, q(x) = \oint d^3\Sigma_\mu K^\mu = N_I \in \mathcal{Z}$$

$q(x)$ is the *topological density* and N_I the *instanton number*.

Applications of Topology

Deep connection between topology and fermions.

- Dirac operator $\not{D}$ has N_I exact chiral 0-modes
- Density of small eigenvalues of $\not{D}$ related to *topological susceptibility*

$$\chi \equiv \frac{1}{\beta V} \left\langle \left(\int d^4x \; q(x) \right)^2 \right\rangle = \int d^4x \; \langle q(x) q(0) \rangle$$

Axion field: explanation for **why** $|\Theta| < 10^{-10}$.
Important role in cosmology, possibly the Dark Matter.
Cares about

- Topological susceptibility $\chi(T)$ up to $T = 1100$ MeV
- Response of topology to E&M fields, especially $q(x)$ in presence of $\epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$

Main questions

- How do we use the lattice to find the topological susceptibility of full QCD up to 1100 MeV temperature?
- Above item requires very fine lattices. How do we determine lattice parameters accurately, especially quark masses?
- How does topology respond to μ_I, and to magnetic fields?
- How does an E&M background with nonzero $\epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} = \vec{E} \cdot \vec{B}$ generate topology?
- How sensitive is topology to heavy (bottom) quarks and to infrared colored fields?

Overview of the projects

Central goal is to get *nonperturbative* information.
Therefore all but the last subproject are *lattice* projects.
In each case, the main challenge is the need for *novel techniques*

- Reweighting
- Gradient-flow scale matching
- Electromagnetic background field methods
- Precise topology on rather coarse lattices with gradient flow
- Continuum, chiral limits particularly challenging

Topology to high temperature

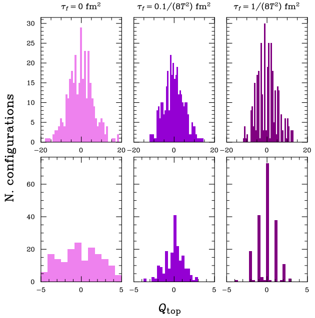
Quenched: lattice differs from perturbation theory by ~ 10.
Need unquenched lattice results

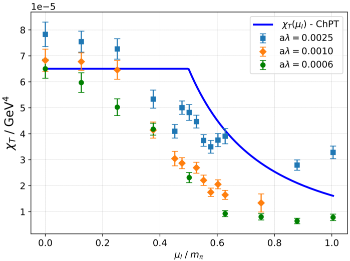
Topology becomes *very rare* at high-T.
Of order 1 lattice configuration in 10^7 will be topological
Markov-chain methods explore in continuous way, but topology means distinct sectors.

- Need to *reweight* to prefer topological configurations by a known amount
- Need to detect lattice states *intermediate* between non-topological and topological
- Need two-stage study: *find correct* amount of reweighting, then *evolve with that reweighting*
- Need to handle significant difference between fermionic spectrum with and without topology
- Requires reweighting in fermion mass parameters

Topology with μ_I and B

μ_I should suppress topology. Effect predicted but never tested.
Limited information about topology with B fields. (One vacuum calculation)
Techniques for generating ensembles with μ_I or B well developed
Because we need low T or vacuum, driven to coarse lattices.
Primary challenges associated with topology on coarse lattices.

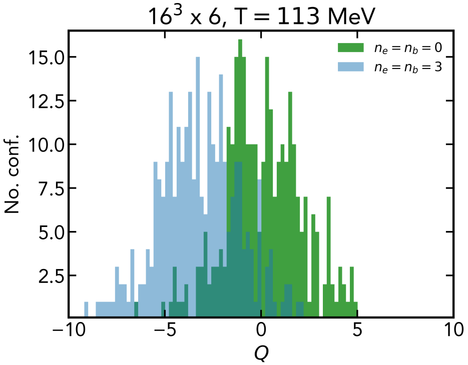




Interaction with electromagnetic fields

Technique to put B field on lattice well developed.
If we use it to put E field in, it's technically imaginary E field.
Creates shift in preferred topology sector.
Origin of interaction between QCD and E&M $\vec{E} \cdot \vec{B}$.

Prediction (Chiral Pert Thy)
Never tested before
Until now!



Analytical considerations: bottom quark

Bottom quark, $m \simeq 4.5$GeV, is

- not heavy enough to ignore at $T = 1100$ MeV
- Heavy enough to make lattice study inconvenient
- Would require extensive scale-setting studies to do 2+1+1+1 flavor lattices.

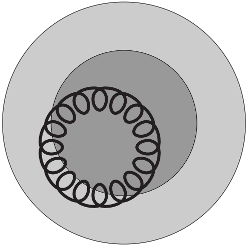
Heavy enough to include its effects perturbatively?
Effect of light quarks in vacuum long established. Also studied:

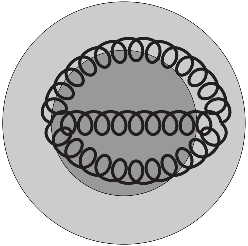
- Massless quarks at finite temperature
- Massive quarks at zero temperature

Need to combine these techniques to do massive + finite temperature.
Easier said than done!

CRC–TR 211

At high temperatures, perturbation should (?) work
Quenched: known factor ~ 10 difference between lattice, pert thy.
Even at $T > 1$ GeV, infrared sector strongly coupled
How does it influence topology? What happens at NLO?
Study effect of *interacting fluctuations* in topological background





Main questions

CRC-TR 211

- How do we use the lattice to find the topological susceptibility of full QCD up to 1100 MeV temperature?
- Above item requires very fine lattices. How do we determine lattice parameters accurately, especially quark masses?
- How does topology respond to μ_I, and to magnetic fields?
- How does an E&M background with nonzero $\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} = \vec{E} \cdot \vec{B}$ generate topology?
- How sensitive is topology to heavy (bottom) quarks and to infrared colored fields?