Progress Report for Project B03

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1 Kinetic theory (classic and quantum)

2 Hydrodynamics theory

3 Hydrodynamics and magnetohydrodynamics simulations

4 Causality and Stability

5 Quantum kinetic theory in curved spacetime

6 Microscopic transport

What the Project Promised

- Quantum kinetic theory for spin-1/2 and spin-1 particles
- Equations of motion of dissipative spin-magnetohydrodynamics (Spin-MHD)
- Causality and Stability of the resulting equations of motion
- Numerical algorithms for solving these equations for heavy-ion collisions (HIC) and astrophysical applications
  - Interface from hydrodynamics to hadronic transport (SMASH as after-burner)
  - Extending the already developed BHAC-QGP code
- Extending the microscopic spin transport (SMASH)
- Applying SMASH and Spin-MHD together to explore polarization signatures in HICs
- Impact of spin dynamics on neutron star properties and binary neutron-star mergers

Goals:

- Derive Boltzmann-type equations from quantum kinetic theory for particles of spin 1/2 and spin 1
- ...including nonlocal collisions
- ...and electromagnetic (EM) fields

Timeline:

2021 Complete derivation of quantum kinetic equation for spin-1/2 particles including non-local collisions and electromagnetic fields

2022 Derive quantum kinetic equation for spin-1 particles including non-local collisions

Quantum kinetic theory: Where Are We?

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- Established spin transport for particles of spin 1/2. ✓

  N. Weickgenannt et al., Phys. Rev. Lett. 127 (2021), 052301

  N. Weickgenannt et al., Phys. Rev. D 104 (2021) 1, 016022

  D. Wagner, N. Weickgenannt, D. H. Rischke, Phys. Rev. D 106 (2022) 11, 116021

- Established spin transport for particles of spin 1. ✓

  D. Wagner, N. Weickgenannt, E. Speranza, Phys. Rev. D 108 (2023) 11, 116017

- Collision effects are included ✓

- Electromagnetic fields are included but further exploration is required

Compared to the planned timeline, we are mostly on time.

Spin hydro: Goals and Timeline

Goals:

- Using the method of moments to derive relativistic dissipative spin magnetohydrodynamics from the kinetic equations

Timeline:

2022 Develop spin hydrodynamics for Dirac fermions

2023 Develop spin hydrodynamics for vector particles, compute transport coefficients for spin-1/2 hydrodynamics

2024 Compute transport coefficients for spin-1 hydro

2025 Complete calculation of transport coefficients

Spin hydro: Where Are We?

- Derived relativistic dissipative spin hydrodynamics for spin-1/2 particles ✓

  N. Weickgenannt, D. Wagner, E. Speranza, D. H. Rischke, Phys. Rev. D 106 (2022) 9, 096014

  N. Weickgenannt, D. Wagner, E. Speranza, D. H. Rischke, Phys. Rev. D 106 (2022) 9, L091901

- Derived relativistic dissipative spin hydro for spin-1 particles ✓

  D. Wagner, N. Weickgenannt, E. Speranza, Phys. Rev. Res. 5 (2023) 1, 013187

- Established the timescale of spin dynamics ✓

  D. Wagner, M. Shokri, D. H. Rischke, in preparation

- Developed an improved method for resumming hydrodynamics ✓

  D. Wagner, A. Palermo, V. E. Ambruș, Phys. Rev. D 106 (2022) 1, 016013

  D. Wagner, L. Gavassino, Phys. Rev. D 109 (2024) 1, 016019

- Used these results to derive resummed spin hydrodynamics ✓(not yet published 🚧)

- EM fields are not yet included in hydro 🚧

Compared to the planned timeline, we are on time.

Causality and Stability: Goals and Timeline

Goals:

- Causality and Stability (CS) of dissipative spin MHD

Timeline:

2021 Complete linear causality and stability analysis of ideal and resistive magnetohydrodynamics for spin-0 particles

2022 Perform linear causality and stability analysis for magnetohydrodynamics of polarizable fluids

2023 Perform linear causality and stability analysis for spin-magnetohydrodynamics for spin-1/2 particle

2024 Perform linear causality and stability analysis for spin-hydrodynamics for spin-1 particle

2025 Complete linear causality and stability analysis

Causality and Stability: Where Are We?

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- SC analysis for dissipative hydro with shear and net charge diffusion for non-zero background charge ✓

  J.Sammet, M. Mayer, D. H. Rischke, Phys. Rev. D107 (2023)

- SC of multicomponent charge diffusion ✓
- Proof of SC of EM sector in IS-type theories of MHD for nonpolarizable fluids ✓

  L. Gavassino, M. Shokri, Phys.Rev.D 108 (2023) 9, 096010

- Linear stability analysis in inhomogeneous EQ configurations ✓

  M. Shokri, D. H. Rischke, Phys.Rev.D 108 (2023) 9, 096029

- Nonlinear causality of first-order Hydro ✓

  F. Bemfica, M. Martinez, M. Shokri, Phys.Rev.D 108 (2023) 5, 5

- Stability and causality analysis for dissipative spin $(\frac{1}{2})$ hd 🚧
- Stability and causality of polarizable fluid 🚧
- Linear waves in resistive dissipative MHD 🚧

Compared to the planned timeline, we are on time.

Goals:

- Numerical implementation of second-order resistive MHD equations to the pre-existing infinite conductivity BHAC-QGP code.
- Extend the already developed numerical infrastructure of BHAC-QGP to solve the equations of relativistic dissipative spin-hydrodynamics.
- Couple SMASH afterburner to BHAC-QGP at freezeout

Timeline:

2021 Implement full coupling between BHAC-QGP and SMASH without consideration of spin

2022 Implement flux-conservative formulation in BHAC-QGP and test it.

2023 Solve spin-magnetohydrodynamics equations to simulate heavy-ion collisions

Spin-(M)HD in simulations: Where Are We?

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- BHAC-QGP: three-dimensional MHD simulations of relativistic heavy-ion collisions ✓

  M Mayer, L Rezzolla, H. Elfner, G. Inghirami, D. H. Rischke preprints: 2403.08668 and 2403.08669 (2024)

- Charge diffusion in relativistic resistive second-order dissipative MHD ✓

  A. Dash, M. Shokri, L. Rezzolla, D. H. Rischke, Phys. Rev. D107 (2022)

  A. Dash, A. K. Panda, Phys.Lett.B 848 (2024) 138342

- Implementation of charge diffusion equation into the BHAC-QGP code.✓
- Interface for SMASH initial conditions to BHAC-QGP code.
- Implementation of equations of spin-hydrodynamics into the framework of BHAC-QGP.
- Implementation of finite μ_Q EoS.
- Numerical computation of polarization in the Navier-Stokes approximation

We are behind compared to the planned timeline.

Microscopic spin transport: Goals and Timeline

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Goals:

- Implement spin degrees of freedom in the transport approach SMASH

Timeline:

2021 Assess angular-momentum conservation in the transport approach

2022 Implement Cooper-Frye particlization transition including spin degree of freedom

2023 Extrapolate treatment of spin to other hadronic species
Implement spin degrees freedom and hyperon cross-sections

2024 Evaluate hadronic-rescattering effects including spin degrees of freedom on polarization observables

2025 Explore adjusting angular distributions of scatterings according to spin polarization of particles

Microscopic spin transport: Where Are We?

- Assessment of angular momentum generation and conservation in SMASH ✓

  Nils Sass, Marco Müller, Oscar Garcia-Montero, and Hannah Elfner, Phys. Rev. C 108, 044903 (2023)

- Inclusion of spin degrees of freedom in SMASH ✓
- Incorporation of spin degrees of freedom in Cooper-Frye particlization transition ✓(still needs to be tested 🚧)
- Evaluation of hadronic rescattering effects including spin degrees of freedom on polarization observables 🚧

Compared to the planned timeline, we are slightly behind.

Binary neutron stars: Goals and Timeline

Goals:

- Carry out simulations of spin-MHD in binary neutron-star mergers
- Simulate the development of the Kelvin-Helmholtz instability (KHI) in these systems

Timeline:

2023 Develop local special-relativistic ideal and resistive MHD simulations of KHI to mimic magnetic-field amplification in neutron-star mergers

2024 Develop local special-relativistic spin-MHD simulations of KHI

2025 Perform a detailed comparison of ideal, resistive, and spin-MHD in the development of KHI

Binary neutron stars: Where Are We?

- Box simulations for special-relativistic KHI in ideal MHD ✓

Compared to the planned timeline, we are behind.

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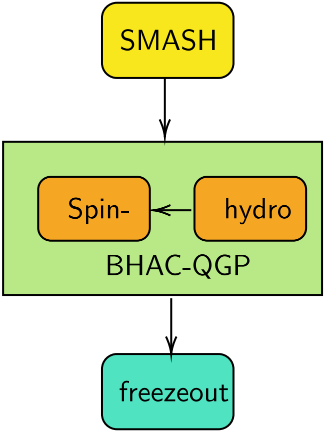
Numerical simulation of spin-hd for HICs



- Feeding initial conditions including vorticity from SMASH to BHAC-QGP
- Implementing spin dynamics in BHAC-QGP (on top of hydro)
- Using LQCD equation of state in BHAC-QGP
- Implementing shear dynamics in BHAC-QGP (important input for spin dynamics)
- Computing $\Lambda - \bar{\Lambda}$ polarization
- Computing vector spin alignment

B03: Outlook

- Quantum kinetic theory
  - Theoretical framework (without EM fields) complete ✓
  - Next step: Include electromagnetic fields 🚧
  - Boltzman-type equations for scalar, spin 1/2, and vector particles in relevant geometries 🚧
- Spin hydrodynamics
  - Theoretical framework (without EM fields) complete ✓
  - Next steps: Inclusion of EM fields & computation of transport coefficients 🚧
  - Theoretical considerations on possible general relativistic applications 🚧
- Causality & Stability
  - Stability of dissipative spin hydrodynamics 🚧
  - Thermodynamic stability of axion MHD 🚧
  - Thermodynamic stability of polarizable fluid 🚧

What the Project Promised (again)

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Appendix

Local Λ polarization

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Solving standard dissipative hydrodynamics and feeding the results into polarization formula of Weickgenannt et al, 2022 on the freezeout surface: preliminary results signal that the standard shear tensor contributes in the right direction [N. Saß, D. Wagner, M.S., H. Elfner, and D.H. Rischke, in progress.]

$$\Pi^{\mu}(p)_{\rm NS} = \int d\Sigma \cdot p \, \frac{f_{0p}}{2\mathcal{N}} \Bigg\{ -\frac{\hbar}{2m} \tilde{\Omega}^{\mu\nu} p_\nu + \left(g^{\mu}_{\nu} - \frac{u^{\mu} p_{\langle \nu \rangle}}{E_p} \right) \times \Big[\mathfrak{e}\chi_{\mathfrak{p}} \left(\tilde{\Omega}^{\nu\rho} - \tilde{\varpi}^{\nu\rho} \right) u_\rho - \chi_{\mathfrak{q}} \mathfrak{d} \beta_0 \sigma_{\rho}{}^{\langle\alpha} \epsilon^{\beta\rangle\nu\sigma\rho} u_\sigma p_{\langle\alpha} p_{\beta\rangle} \Big] \Bigg\}$$

Stability of rigidly moving fluids

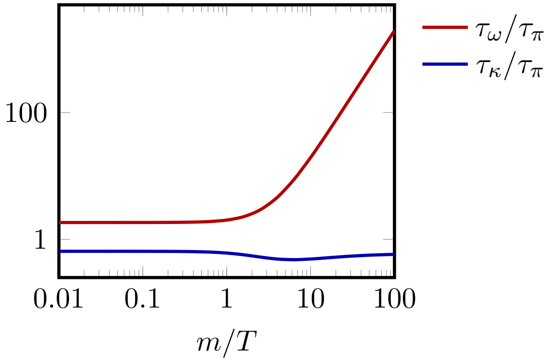
- Does hydrodynamics stay stable for a rapidly rotating fluid?
- In Shokri et al, 2023 we approached the problem of linear stability in inhomogeneous equilibrium configurations (i.e., rigid rotation and pure acceleration), using a mathematical construct that was implicitly introduced by Fonarev.
- In hydrodynamics, this approach leads us to find local dispersion relations, i.e., frequencies that are spacetime-dependent
- Application of our method to MIS hydrodynamics shows that the standard stability conditions work **as long as the system is large enough** compared to the microscopic length scales
- The most vortical fluid $(10^{22}\mathrm{s}^{-1})$ **is still not rotating fast enough** for the boundary effects to be important

Why spin hydrodynamics?

Sign puzzle seems to have been solved but . . .

- **It is not clear how and when the spin degrees of freedom are equilibrated**
- No transport coefficient: hadron polarization is merely kinematic
- Our stance: To describe spin dynamics properly, one would need to extend standard dissipative hydrodynamics to a theory of spin hydrodynamics

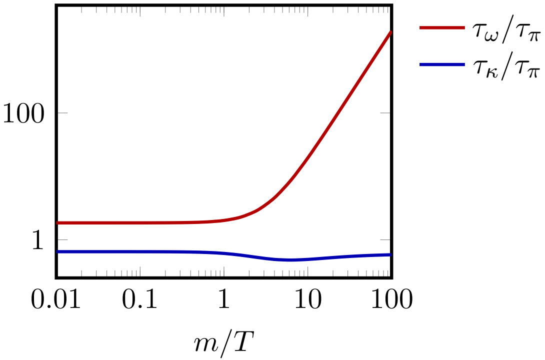
Timescales in spin hydrodynamics



D. Wagner, M. Shokri, D.H. Rischke, in preparation

- Estimation for four-fermion interaction, $\mathcal{L}_{\text{int}} \coloneqq G(\bar{\psi}\psi)^2$
- There are two coupled modes
- Relaxation times larger than typical dissipative timescale τ_π

Timescales in spin hydrodynamics



- Spin degrees of freedom relax quite fast in high-energy collisions
- . . . while these timescales for low-energy collisions might be even larger than the lifetime of the fireball
- A possible explanation of why the results of Becattini et al., 2021 are consistent with data