A07

Transport properties

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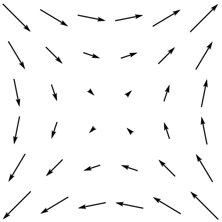
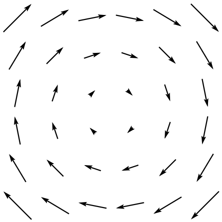
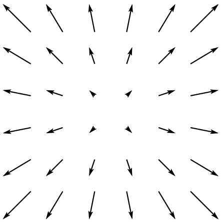
Heavy ions are “large” systems ⇒ locally near equilibrium

$$\partial_\mu T^{\mu\nu} = 0 \qquad \text{Energy-momentum conservation}$$
$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + P\eta^{\mu\nu} \qquad \text{in equilibrium}$$

Equilibrium better approx than free-streaming, but need corrections due to flow derivative. $\partial^\mu u^\nu$: traceless symmetric, antisymmetric, trace:


Shear flow $\sigma^{\mu\nu}$


Vorticity $\Omega^{\mu\nu}$


Divergence Θ

Hydrodynamics Review

Corrections due to gradients:

$$T^{\mu\nu} = T^{\mu\nu}_{\text{equil}} + \pi^{\mu\nu}$$
$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \zeta\Delta^{\mu\nu}\Theta - \eta\tau_\pi u^\alpha\partial_\alpha\sigma^{\mu\nu} + \lambda_3\Omega^\mu{}_\alpha\Omega^{\nu\alpha} + \ldots$$

Represent corrections away from equilibrium due to flow.
Coefficients η, ζ, τ_π, $\lambda_3 \ldots$ from experiment?
Or try to determine them from *first principles*?
Additional coefficients: diffusion for light, heavy quarks;
sphaleron rate for chiral imbalance of lightest quarks

First principles determination is central goal of this project
Perturbative techniques and (mostly) nonperturbative lattice approach.

Main questions

Lattice allows nonperturbative evaluation.
Complicated by need to *analytically continue*
Complicated by need for noise reduction technique: *Gradient Flow*

Main questions we want to answer:

- How to use gradient flow with fermions
- Unquenched lattice transport coefficients with gradient flow
- Sphaleron rate from the Euclidean lattice
- Transport and vorticity

Relate transport coefficients to correlation functions (Kubo)

$$\eta = \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega} = \lim_{\omega \to 0} \frac{1}{\omega} \int d^4 x e^{i\omega t} \left\langle \left[T_{xy}(x) \, , \, T_{xy}(0) \right] \right\rangle$$

Relate correlation function to Euclidean function

$$G_{\mathrm{E}}(\tau) \equiv \int d^3 x \frac{\mathrm{Tr}\, T_{xy}(x) e^{-\tau H} T_{xy}(0) e^{-(\beta - \tau) H}}{\mathrm{Tr}\, e^{-\beta H}}$$

$$G_{\mathrm{E}}(\tau) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\rho(\omega)}{\omega} \frac{\omega \cosh(\omega(\tau - \beta/2))}{\sinh(\omega\beta/2)}$$

Measure Euclidean function, solve above inverse relation

Noise reduction: compute Euclidean function using gradient flow.

Lattice, Gradient Flow, Fermions

1'st funding period: performed in pure-glue QCD.
2'nd funding period: need to "unquench". Challenges:

- How should one implement fermions to utilize gradient flow?
- Stress tensor has two terms. Nonperturbative renormalization?

$$T_{xy} = Z_1 F_{x\mu} F_{y\mu} + Z_2 \bar{\psi} \left(\gamma_x D_y + \gamma_y D_x \right) \psi$$

- Various connected, disconnected contributions to correlator

Need technical developments to overcome these issues.
Then we can apply to η, ζ, topology with known methods.

Unquenched lattice heavy quarks

Integrate out heavy quarks to replace with *Wilson line*
Diffusion coefficient determined by force-force correlator

$$\int dt \, \langle F_i(t) F_i(0) \rangle = \langle (E_i + F_{ij} v_j)(E_i + F_{ij} v_j) \rangle = \langle E_i E_i \rangle + \frac{\langle v^2 \rangle}{3} \langle F_{ij} F_{ij} \rangle$$

New in this funding period:

- Computing $F_{ij} F_{ij}$ contribution
- Handling operator normalization for these magnetic terms
- Unquenching – no technical obstacles, just lower statistics

Sphaleron rate, saddles, lattice

Sphaleron rate – *real-time* rate of topology change
Previously computed at *weak coupling* via classical statistical real-time lattice techniques (similar to A06)
What about at strong coupling?

Rate controlled by saddlepoint.
Saddle – something we can find on the lattice
New technique to find saddlepoint-controlled rates from the lattice

Transport and vorticity

Vorticity: $\Omega_{ij} = \partial_i v_j - \partial_j v_i$ "rotation"
Antisymmetric tensor: $T^{\mu\nu}$ can only depend quadratically
Kubo relation:

$$T_{xy} = \ldots + \lambda_3 \Omega_{xz} \Omega_{yz}$$

$$\lambda_3 \propto \lim_{\vec{k} \to 0} \frac{\partial^2}{\partial k_z^2} \int d^4 r \, e^{ik\cdot r} \left\langle \left[T_{x0}(r) \, , \, T_{y0}(0) \right] \right\rangle$$

No frequency derivative: evaluated straight at $\omega = 0$

$$\lambda_3 \propto \int_0^\beta d\tau \int d^3 r \, r_z^2 \langle T_{0x}(r,\tau) T_{0y}(0,0) \rangle_{\text{Eucl.}}$$

Allows *direct* lattice evaluation *without* reconstruction or analytical continuation

Main questions

- Gradient flow with fermions
- Unquenched lattice transport coefficients with gradient flow
- Sphaleron rate from the Euclidean lattice
- Transport and vorticity