Plan for the round table discussion on transport coefficient

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Transport coefficient:

Response of the system under the external perturbation,

1. Diffusion Coefficient:

$$J = -D\nabla n$$

2. Viscosity:

$$T_{ij} = P\,\delta_{ij} - \eta\,(\nabla_i\, u_j + \nabla_j\, u_i - \frac{2}{3}\delta_{ij}\nabla_k\, u_k) - \zeta\,\delta_{ij}\nabla_k\, u_k$$

Important for hydrodynamic evolution:

$$\nabla_\mu J^\mu = 0$$

$$\nabla_\mu\, T^{\mu\nu} = 0$$

Methods for Estimation

- Perturbation theory: High T limit ($T \gg T_c$).
- SMASH approach: $T < T_c$ in the hadronic phase.
- Lattice QCD technique: In principle valid in all temperature range.
- Constrains from experimental data.
- Calculation using models (NJL/PNJL)

Each of these methods has there own systematic uncertainty.

Perturbation theory

- Kinetic theory description of a system:

$$\left[\frac{\partial}{\partial t} + \vec{v_p} \cdot \frac{\partial}{\partial \vec{x}}\right] f_0^a(\vec{p}, \vec{x}, t) = -(\mathcal{C}[f_1^a])(\vec{p}, \vec{x}, t) \qquad f^a = f_0^a + f_0^a(1 - f_0^a) f_1^a$$

- Solve Boltzmann equation using a variational approach:

$$S^a(p) = (Cf_1)^a(p) \quad \rightarrow \quad \mathcal{Q}[\chi] = (f_1, S) - \frac{1}{2}(f_1, \mathcal{C} f_1)$$

- Transport coefficients can be calculated as:

$$\eta = \frac{2}{15}\mathcal{Q}_{max}\,, \qquad \sigma = \frac{2}{3}\mathcal{Q}_{max}\,, \qquad D_\alpha = \frac{2}{3}\mathcal{Q}_{max}\left(\frac{\partial n_\alpha}{\partial \mu_\alpha}\right)$$

SMASH

- Use infinite matter simulations in equilibrium (box with periodic boundary conditions
- Calculate transports coefficients using the Green-Kubo approach from initial fluctuation:

$$\eta = \frac{V}{T} \int \langle T^{ij}(t) T^{ij}(0) \rangle dt$$
$$\kappa_{ij} = \frac{V}{T} \int \langle J_i(t) J_j(0) \rangle dt$$

- Gives access to transport coefficients as function of temperature and μ_i
- Depends on the interactions in the hadron gas

Lattice Estimates

- $$O_t = \lim_{\omega \to 0} \frac{\rho_O(\omega)}{\omega}$$
- $$G_O^E(\tau, \vec{k}) = \int_0^\infty \frac{d\omega}{\pi} \rho_O(\omega, \vec{k}) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$
- Need good S/N ratio.
- Need renormalization on lattice before continuum extrapolation. (Gradient flow)
- Numerically ill-posed problem. Small number of data points and statistical errors.
- Spectral reconstruction by modeling the spectral function on the basis of physics arguments.

The Plan for the Discussion

- 14:30 to 16:00: 45-minute presentations followed by a 45 min discussion.
- 16:30 to 18:30: Discussion on PQCD estimates and Lattice QCD estimates.
- (Tomorrow) 9:30 to 10:30: Shear viscosity from phenomenology and Experiment.
- (Tomorrow) 14:00 to 16:00: Input on the too difficult questions with inputs from PI. Any left-out topic.
- (Tomorrow) 16:30 to 17:30: Summary and preparation of report.