

# Plan for the round table discussion on Transport Coefficients

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# Transport coefficient:

Response of the system under the external perturbation,

① Diffusion Coefficient:

$$J = -D\nabla n$$

② Viscosity:

$$T_{ij} = P \delta_{ij} - \eta (\nabla_i u_j + \nabla_j u_i - \frac{2}{3} \delta_{ij} \nabla_k u_k) - \zeta \delta_{ij} \nabla_k u_k$$

Important for hydrodynamic evolution:

$$\nabla_\mu J^\mu = 0$$

$$\nabla_\mu T^{\mu\nu} = 0$$

# Methods for Estimation

- Perturbation theory: High  $T$  limit ( $T \gg T_c$ ).
- SMASH approach:  $T < T_c$  in the hadronic phase.
- Lattice QCD technique: In principle valid in all temperature range.
- Constrains from experimental data.
- Calculation using models (NJL/PNJL)

Each of these methods has their own systematic uncertainty.

## Perturbation theory

- Kinetic theory description of a system:

$$\left[ \frac{\partial}{\partial t} + \vec{v}_p \cdot \frac{\partial}{\partial \vec{x}} \right] f_0^a(\vec{p}, \vec{x}, t) = -(\mathcal{C}[f_1^a])(\vec{p}, \vec{x}, t) \quad f^a = f_0^a + f_0^a(1 - f_0^a)f_1^a$$

- Solve Boltzmann equation using a variational approach:

$$S^a(p) = (\mathcal{C}f_1)^a(p) \quad \rightarrow \quad \mathcal{Q}[f_1] = (f_1, S) - \frac{1}{2} (f_1, \mathcal{C}f_1)$$

- Transport coefficients can be calculated as:

$$\eta = \frac{2}{15} Q_{max}, \quad \sigma = \frac{2}{3} Q_{max}, \quad D_\alpha = \frac{2}{3} Q_{max} \left( \frac{\partial n_\alpha}{\partial \mu_\alpha} \right)$$

- Use infinite matter simulations in equilibrium (box with periodic boundary conditions)
- Calculate transports coefficients using the Green-Kubo approach from initial fluctuation:

$$\eta = \frac{V}{T} \int \langle T^{ij}(t) T^{ij}(0) \rangle dt$$
$$\kappa_{ij} = \frac{V}{T} \int \langle J_i(t) J_j(0) \rangle dt$$

- Gives access to transport coefficients as function of temperature and  $\mu_i$
- Depends on the interactions in the hadron gas

## Lattice Estimates

$$O_t = \lim_{\omega \rightarrow 0} \frac{\rho_O(\omega)}{\omega}$$

$$G_O^E(\tau, \vec{k}) = \int_0^\infty \frac{d\omega}{\pi} \rho_O(\omega, \vec{k}) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

- Need good signal-to-noise ratio.
- Need renormalization on lattice before continuum extrapolation. (Gradient flow)
- Numerically ill-posed problem. Small number of data points and statistical errors.
- Spectral reconstruction by modeling the spectral function on the basis of physics arguments.

# The Plan for the Discussion

## Today (Room SR 9):

- 14:30 to 16:00: Presentations (45 minutes) followed by discussions (45 minutes).
- 16:30 to 18:30: Discussion on PQCD estimates and Lattice QCD estimates.

## Tomorrow:

- 9:30 to 10:30: Shear viscosity from the Phenomenology and Experiment.
- 14:00 to 16:00: Input on the too difficult questions with inputs from Pls. Any left-out topic.
- 16:30 to 17:30: Summary and preparation of report.