Heavy Quark: Transport coefficient

1. Time scale for diffusion $\frac{M}{T^2} \gg \frac{1}{T}$
   Langevin Equation:

$$\frac{dx}{dt} = \frac{p}{m}$$
$$\frac{dp}{dt} = \zeta(t) - \eta_d\, p$$
$$\langle \zeta_i(t)\zeta_j(t')\rangle = \kappa\delta_{ij}\,\delta(t-t')$$
$$D = \frac{T}{\eta_d\, M} \qquad \text{Einstein relation}$$

Simon Caron-Huot et al, JHEP 04 (2009) 053

1. This leads to a transport peak in the in vector current correlator:

$$\rho_{ii} = 3\chi_q \frac{T}{M}\frac{\omega\,\eta_d}{\omega^2+\eta_d^2}$$
$$D = \frac{1}{3\chi_q}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega)}{\omega}$$

2. Extremely narrow transport contribution $\omega \sim \frac{T^2}{M}$.
   For light quark transport contribution $\omega \sim g^4 T$
   Bound state contribution $\omega \sim 2M$.

Heavy Quark: Transport coefficient

1. Time scale for diffusion $\frac{M}{T^2} \gg \frac{1}{T}$
   Langevin Equation:

$$\kappa = \frac{1}{3}\int_{-\infty}^{\infty} \langle \zeta(t)\zeta(0)\rangle$$

Fluctuation-Dissipation theorem:

$$\eta_d = \frac{\kappa}{2M_q T}$$

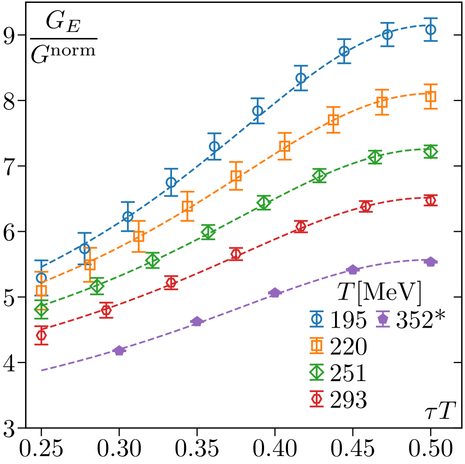
1. Calculate in the expansion of $1/M_q$

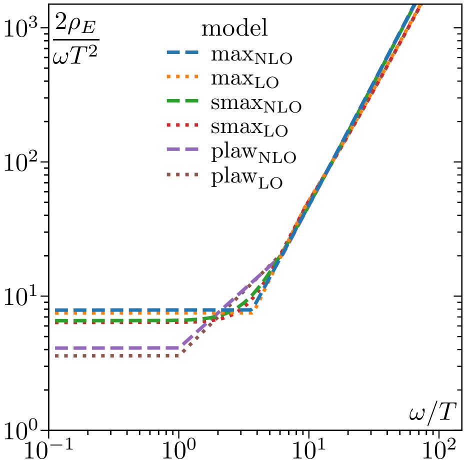
$$\kappa = \kappa_E + v^2\kappa_B; \kappa_O = \lim_{\omega\to zero} \frac{\rho_O(\omega)}{\omega}$$

2. ρ_O is the spectral function of G_O

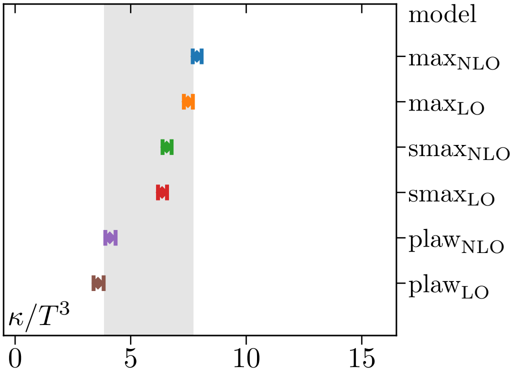
$$G_O(\tau) = -\frac{\mathrm{ReTr}(U(\beta,\tau)O(\tau)U(\tau,0)O(0))}{3\mathrm{Tr}(U(\beta,0))}$$

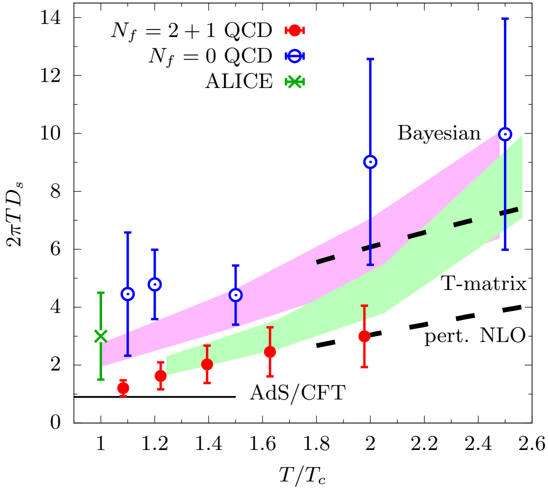
3. These correlatosr renormalization. $O = B$ the renormalization procedure is tricky.
   M. Laine et al, arXiv: JHEP06(2021)139 [hep-lat]
   L. Altenkort et al, arXiv:2402.09337 [hep-lat]



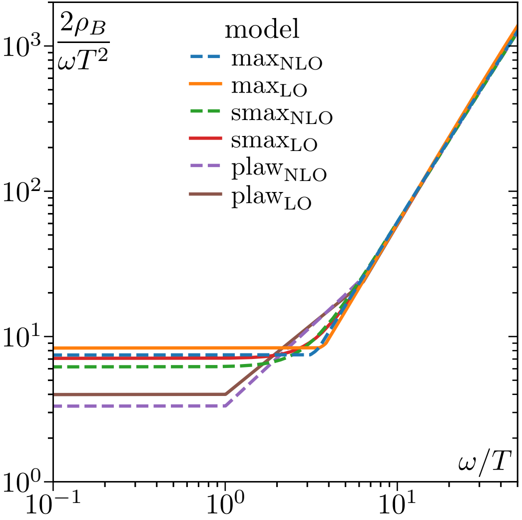


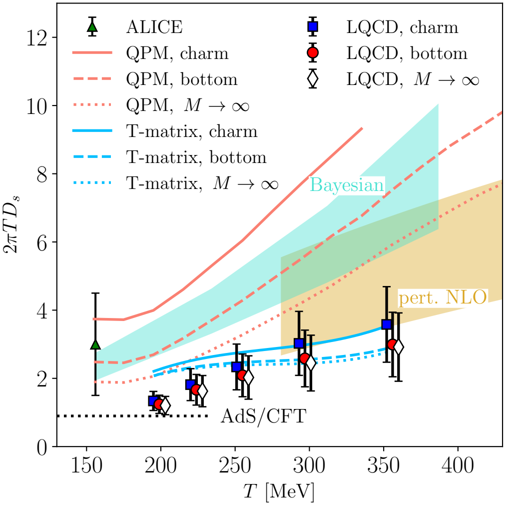
L. Altenkort et al, PRL.130.231902





L. Altenkort et al, PRL.130.231902





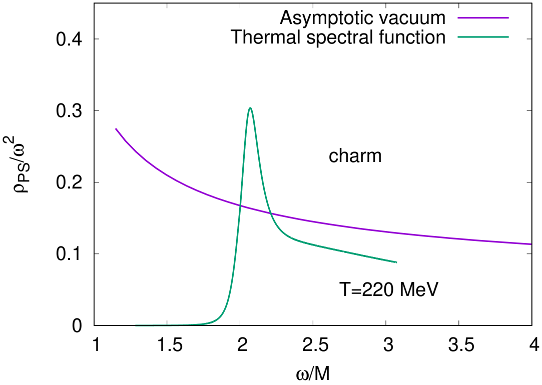
L. Altenkort et al, PRL.132.051902

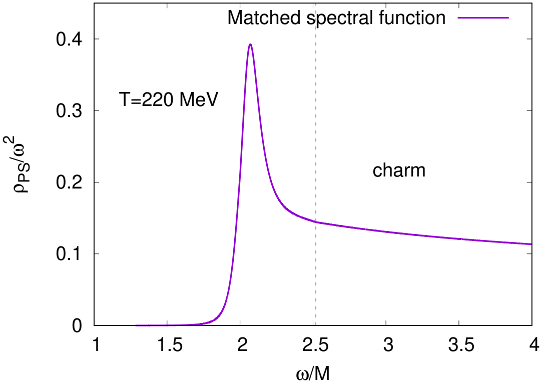
Transport from vector channel

Spectral function in different region

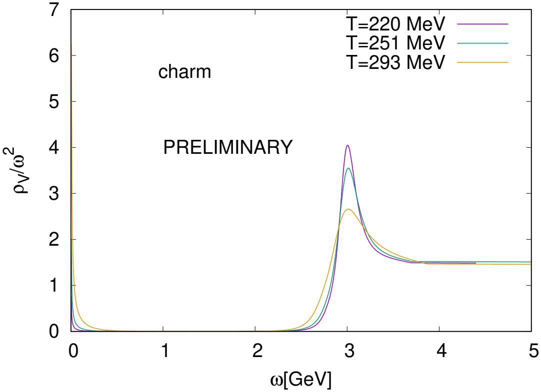
- $\omega \gg 2M$
  Thermal effects are suppressed. Vacuum perturbation theory will work.
- $\omega \sim 2M$
  Thermal effects are important. Spectral function needs to be calculated using thermal potential.
- $\omega \ll 2M$
  For the vector channel, there is a contribution around $\omega \sim 0$ due to transport.

$$\rho_{PS}(\omega) = A_0\, \rho_{PS}^{T}(\omega)\theta(\omega_0 - \omega) + \rho_{PS}^{T=0}(\omega)\theta(\omega - \omega_0)$$

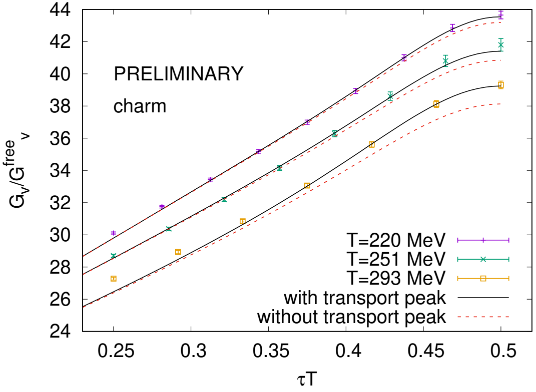




- $A_0 \sim 0.88\, M - 1.2\, M$
- $\omega_0 \sim 2\, M - 3\, M$



T=220 MeV
T=251 MeV
T=293 MeV
charm
PRELIMINARY
ρ_V/ω^2
ω[GeV]
0
1
2
3
4
5
6
7



PRELIMINARY
charm
G_V/G^{free}_V
τT
T=220 MeV
T=251 MeV
T=293 MeV
with transport peak
without transport peak
0.25
0.3
0.35
0.4
0.45
0.5

- Large contribution from UV/bound state region.
- T-L for heavy quarks? can it suppress boundstate region?
- How does gradient flow affect the the bound state region?